Spacetime curvature and Higgs stability during and after inflation

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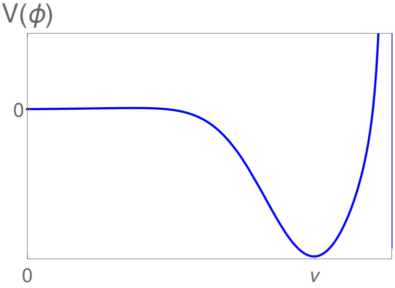
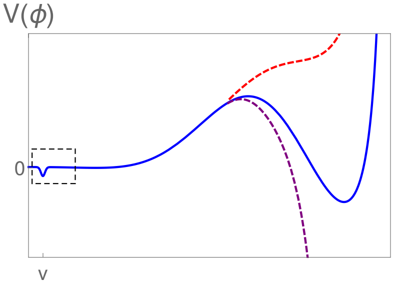
Outline

Standard Model Higgs potential



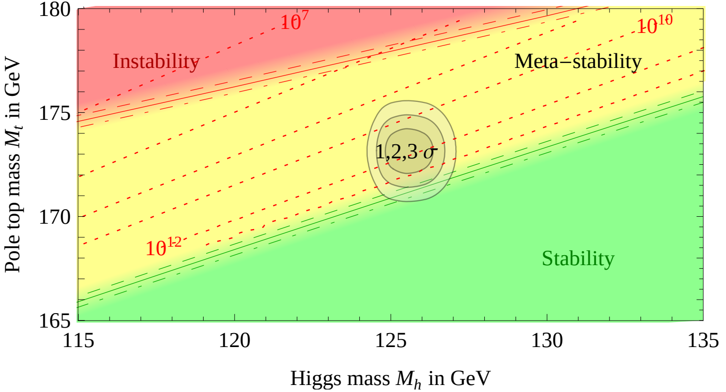


- $V(\phi)$ has a minimum at $\phi = v$
- Behaviour very sensitive to M_h and M_t
- A vacuum at $\phi \neq v$ incompatible with observations

New physics needed to stabilize the vacuum?

Current status

Figure : Degrassi et al. (2013)

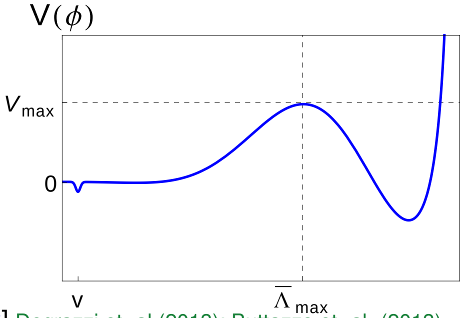


- *Meta*stable at 99% CL [1]
    - Lifetime much longer than $13.8 \cdot 10^9$ years
- Is this also true for the early Universe ?

[1] Buttazzo et al. (2013); Spencer-Smith (2014); Bednyakov, Kniehl, Pikelner, & Veretin (2015)

Inflation and the Standard Model

- We assume the SM to be valid at high energies
  - Potential peaks at $\overline{\Lambda}_{\rm max}$
- Assuming also an early stage of exponential cosmological expansion (inflation) with a scale H
  - Important if $\overline{\Lambda}_{\rm max} \lesssim H$
  - State of the art calculations [2]: $\overline{\Lambda}_{\rm max} \sim 10^{11}\text{GeV}$



BICEP2/Keck/Planck

$H \lesssim 10^{14}\text{GeV}$

BICEP2:

$\overline{\Lambda}_{\rm max} \ll H$

[2] Degrazzi et. al.(2013); Buttazzo et. al. (2013)

Outline

Higgs stability during inflation

- Inflation induces fluctuations to the Higgs field $\Delta\phi \sim H$
- Fluctuations may be treated as stochastic variables [3]

$\Rightarrow$ We can assign a probability density $P(\phi)$ to ϕ

- The essential input for $P(\phi)$ is $\bar{V}_{\rm eff}(\phi)$, the *effective potential*

[3] Starobinsky (1986); Starobinsky & Yokoyama (1994)

1-loop Effective potential

- Derivation of $V_{\text{eff}}(\phi)$ is a standard calculation [4]
- A theory with a massive self-interacting scalar field

$$V_{\text{eff}}(\phi) = \underbrace{\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4}_{\text{classical}}$$

$$+ \underbrace{\frac{M(\phi)^4}{64\pi^2}\left[\log \overbrace{\left(\frac{M(\phi)^2}{\mu^2}\right)}^{\text{effective mass}} - \frac{3}{2}\right]}_{\text{quantum}} \quad ; M(\phi)^2 = m^2 + \frac{\lambda}{2}\phi^2$$

- μ is the *renormalization scale*
- Similarly one may derive the potential for the SM Higgs

[4] Coleman & Weinberg (1972)

Effective potential for the SM Higgs

$$V_{\rm eff}(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \sum_{i=1}^{5}\frac{n_i}{64\pi^2}M_i^4(\phi)\left[\log\frac{M_i^2(\phi)}{\mu^2} - c_i\right]$$

$$; M_i^2(\phi) = \kappa_i\phi^2 - \kappa_i'$$

Φ	i	n_i	κ_i	κ_i'	c_i
$W^{\pm}$	1	6	$g^2/4$	0	$5/6$
Z^0	2	3	$(g^2+g'^2)/4$	0	$5/6$
t	3	-12	$y_t^2/2$	0	$3/2$
ϕ	4	1	3λ	m^2	$3/2$
χ_i	5	3	λ	m^2	$3/2$

- Explicit μ dependence?

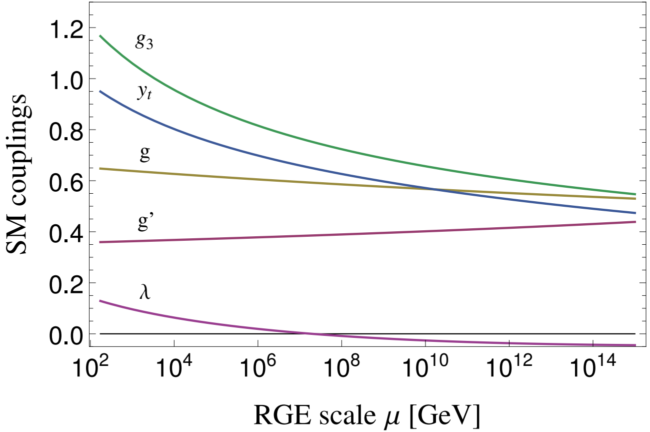
Callan-Symanzik equation for massless $\lambda\phi^4$ theory

- The effective potential is renormalized at a scale μ
  $\lambda_0 \to \lambda_R + \delta\lambda, \quad \phi \to (1+\delta Z)\phi$
- However, the physical result must not depend on μ
- We can impose this by demanding

$$\frac{d}{d\mu} V_{\text{eff}}(\phi) = 0$$

- This can be used to improve the perturbative result
- Leads to *running parameters*, e.g. $\lambda(\mu)$
- Same can be done for the SM

SM running (1-loop)



- For large ϕ, the potential is dominated by the quartic term $\lambda\phi^4$

$$V(\phi) \sim \frac{\lambda(\mu)}{4}\phi^4$$

Scale independence V_{eff}

- One can easily show that for the SM to 1-loop [5]

$$\frac{d}{d\mu}\bar{V}_{\text{eff}} = 0 + \mathcal{O}(\hbar^2)$$

- We must choose μ to make the higher order terms as small as possible [6]

The optimal choice

$$\mu \sim \phi$$

$\Rightarrow$ No large logarithms

- Now we have a well-defined potential with no unknown parameters!

[5] Casas et. al. (1994)
[6] Ford et. al. (1993)

Generalization to curved space

<2->

- It is possible to include (classical) gravity in the quantum calculation, $R = 12H^2$

$\Rightarrow$ The SM includes a non-minimal ξ-term, $\sim \xi R\phi^2$

  - Always generated by running in curved space
  - Virtually unbounded by the LHC, $\xi_{\rm EW} < 10^{15}$ [7]

- Curvature induces running of the constants [8]
- Leading potential contributions:

Flat space, $\phi \gg m$

$$V_{\rm eff}(\phi) \approx \frac{\lambda(\phi)}{4}\phi^4$$

Curved space, $H \gg \phi \gg m$

$$V_{\rm eff}(\phi) \approx \frac{\lambda(H)}{4}\phi^4 + \frac{\xi(H)}{2}R\phi^2$$

[7] Atkins & Calmet (2012)
[8] Zurek, Kearney & Yoo (2015); TM (2014)

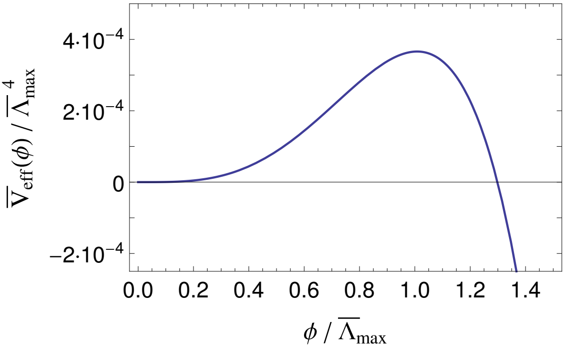
1-loop Effective potential in curved space

$$V_{\text{eff}}(\phi, R) = -\frac{1}{2}m^2(t)\phi(t)^2 + \frac{1}{2}\xi(t)R\phi(t)^2 + \frac{1}{4}\lambda(t)\phi(t)^4$$

$$+\sum_{i=1}^{9}\frac{n_i}{64\pi^2}M_i^4(t)\left[\log\frac{|M_i^2(t)|}{\mu^2(t)} - c_i\right] \qquad ; M_i^2(t) = \kappa_i\phi(t)^2 - \kappa_i' + \theta_i R$$

Φ	i	n_i	κ_i	κ_i'	θ_i	c_i
$W^\pm$	1	2	$g^2/4$	0	$1/12$	$3/2$
	2	6	$g^2/4$	0	$-1/6$	$5/6$
	3	-2	$g^2/4$	0	$-1/6$	$3/2$
Z^0	4	1	$(g^2+g'^2)/4$	0	$1/12$	$3/2$
	5	3	$(g^2+g'^2)/4$	0	$-1/6$	$5/6$
	6	-1	$(g^2+g'^2)/4$	0	$-1/6$	$3/2$
t	7	-12	$y_t^2/2$	0	$1/12$	$3/2$
ϕ	8	1	3λ	m^2	$\xi - 1/6$	$3/2$
χ_i	9	3	λ	m^2	$\xi - 1/6$	$3/2$

Stability (Flat)



- For large H ($\sim 10^3\overline{\Lambda}_{\rm max}$), the SM is not stable [9]
- Coupling the Higgs to an inflaton $\sim \Phi^2\phi^2 \Rightarrow$ stable [10]

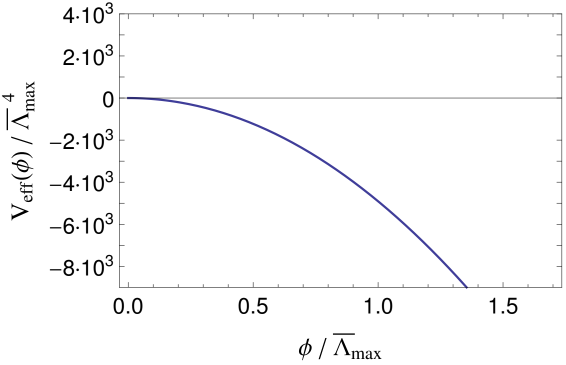
How does including curvature change this?

[9] Kobakhidze & Spencer-Smith (2014); Hook et. al. (2014); Fairbairn & Hogan (2014); Enqvist, Meriniemi & Nurmi (2014); Zurek, Kearney & Yoo (2015)
[10] Lebedev (2012); Lebedev & Westphal (2013)

Stability (curved) I

- First attempt, set $\xi_{\rm EW} = 0$ and $H \sim 10^3 \overline{\Lambda}_{\rm max}$

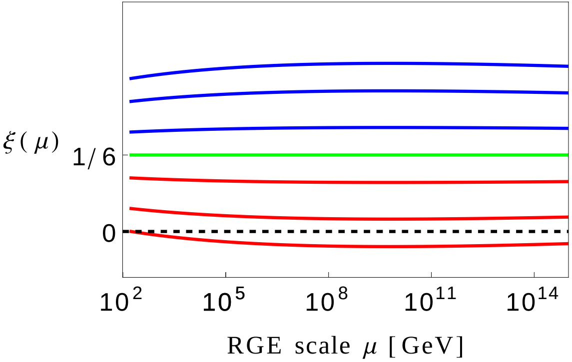
$$V_{\rm eff}(\phi) \approx \frac{\lambda(\mu)}{4}\phi^4 + \frac{\xi(\mu)}{2}R\phi^2$$



- For large H one has $\lambda(\mu) < 0$, since $\mu^2 = \phi^2 + R$
- ξ Can become positive or negative depending on $\xi_{\rm EW}$

Stability results (curved space) II

- For large H one has $\lambda(\mu) < 0$, since $\mu^2 = \phi^2 + R$
- ξ Can become positive or negative depending on $\xi_{\rm EW}$

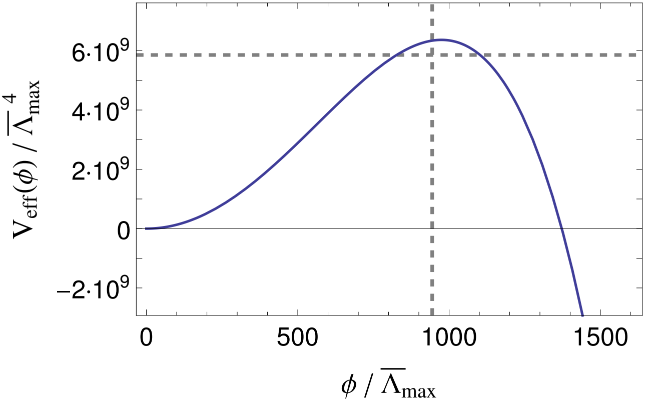


ξ_{EW}

$0,\ 0.05,\ 0.12,\ 1/6,$
$0.22,\ 0.28,\ 0.33$

Stability results (curved space) III

- Now choosing $\xi_{\mathrm{EW}} = 0.1$ [11]



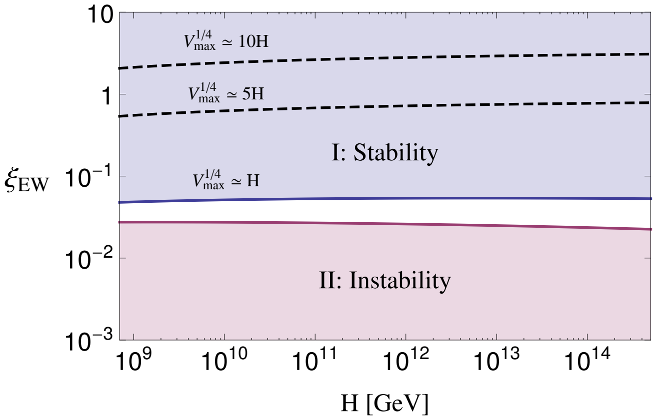
- $V_{\mathrm{max}}(\mathrm{curved}) \gg V_{\mathrm{max}}(\mathrm{flat})$ (and at a higher scale)

$$P \sim \exp\left[-8\pi^2\,(V_{\mathrm{max}}/3H^4)\right] \quad \Rightarrow \quad \text{Stable!}$$

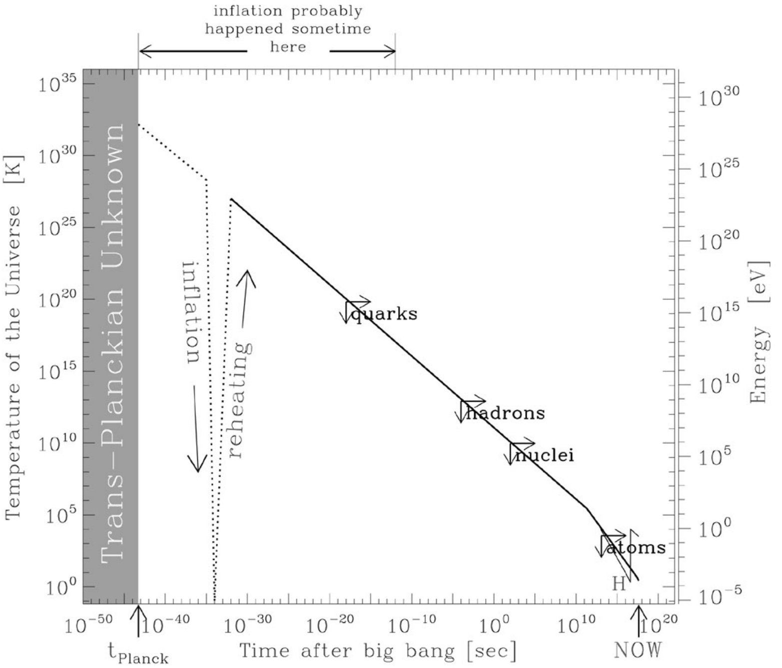
[11] Espinosa, Giudice & Riotto (2008)

Stability results (curved space) IV

- The (in)stability of the potential is determined by ξ_{EW}



Outline


inflation probably
happened sometime
here
Trans−Planckian Unknown
inflation
reheating
quarks
hadrons
nuclei
atoms
H
Temperature of the Universe [K]
Energy [eV]
Time after big bang [sec]
t_{Planck}
NOW
10^{35}
10^{30}
10^{25}
10^{20}
10^{15}
10^{10}
10^{5}
10^{0}
10^{-5}
10^{-50}
10^{-40}
10^{-30}
10^{-20}
10^{-10}
10^{20}

Reheating

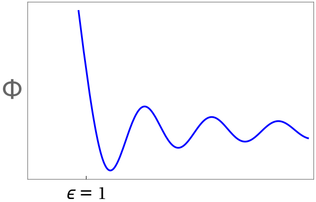
- Equation of state $w = p/\rho$ changes, $w_{\text{inf}} = -1 \to w_{\text{reh}}$
- Energy of inflation is transferred to SM degrees of freedom, which (eventually) thermalize $T = 0 \to T_{\text{reh}}$
- The crucial moment is right after inflation, but *before* thermalization
- A very complicated and dynamical process [12]
  - Reheating $\Leftrightarrow$ *Pre*heating

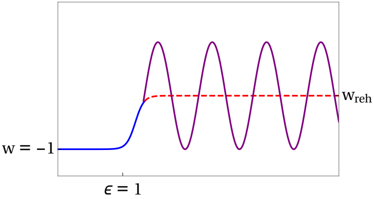
> - The Higgs always feels the dynamics of reheating
>
> (even without a direct coupling to the inflaton)

[12] Kofman, Linde & Starobinsky (1997)

Reheating

- During reheating the inflaton oscillates ($p = w\rho$)





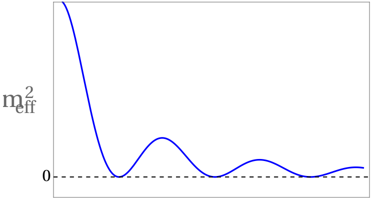
- The inflaton influences the Higgs via gravity

$\Rightarrow$ New stability constraints !

- Two effects:
  - A rapid drop in w, *on average*
  - Oscillations in the complete solution

Oscillating mass (example)

- For example for a coupling $\mathcal{L}_{\rm int} \propto g\Phi^2\phi^2$



Oscillating mass for Higgs

$$m_{\rm eff}^2 \sim g\Phi_0^2\cos^2(t\,M_{\rm inf})$$

- *Parametric resonance* via the Mathieu equation

$$\frac{d^2f(z)}{dz^2} + \Big[A_{\mathbf{k}} - 2q\cos(2z)\Big]f(z) = 0, \qquad z = t\,M_{\rm inf}$$

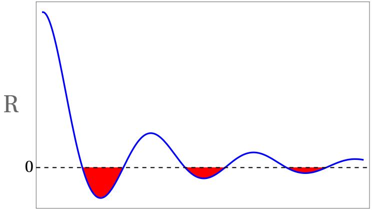
$\Rightarrow$ Exponential amplification
  - May result in a very large fluctuation [13]

[13] Kofman, Linde & Starobinsky (1997)

Oscillating R

- The curvature oscillates during reheating

$$G_{\mu\nu} = \frac{1}{M_{\rm pl}^2} T_{\mu\nu} \quad \Rightarrow \quad R = \frac{1}{M_{\rm pl}^2}\left[4V_{\rm inf}(\Phi) - \left(\frac{d\Phi}{dt}\right)^2\right]$$



Curvature mass ξR oscillates to negative values

- *Tachyonic resonance* [14]
- Oscillations of R via ξ provide efficient reheating
  - *Geometric reheating* [15]

[14] Kofman, Dufaux, Felder, Peloso & Podolsky (2006)
[15] Bassett & Liberati (1997)

Fluctuations from parametric resonance

- Resonance may give large fluctuations,
  $\Rightarrow$ Instabilities ?!
- After *one* oscillation

$$n \sim \exp\left\{\sqrt{\xi}\right\}$$

Superhorizon modes, $\mathbf{k} < aH$

$$\Rightarrow \quad \Delta\phi^2 \sim \left(\frac{H}{2\pi}\right)^2 \frac{\exp\{\sqrt{\xi}\}}{\sqrt{\xi}}$$

- Potentially a huge effect, $\Delta\phi \gg \Lambda_I$

- However, the resonance may be shut off by backreaction

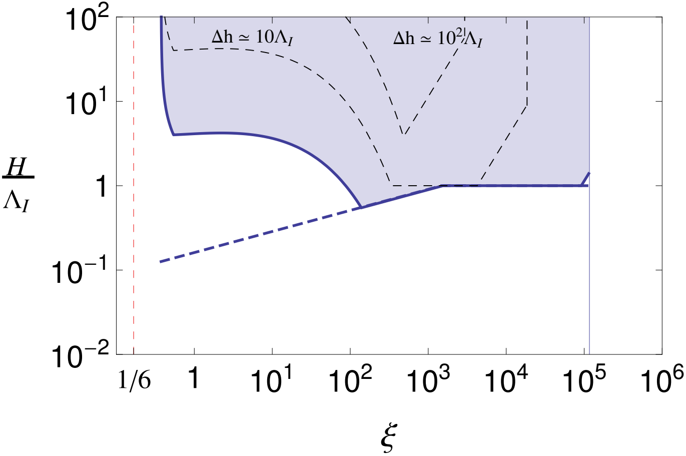
Self-interactions

$$\lambda\langle\hat{\phi}^2\rangle \ll \xi R, \quad \text{if} \quad \lambda > 0$$

Gravity

$$\rho_{\text{Higgs}} \ll 3M_{\text{pl}}^2 H^2$$

Stability results, reheating



$\Rightarrow$ For $H \gtrsim \Lambda_I \sim 10^{11}$GeV, ξ is constrained to be $\sim 1/6$

Outline

Conclusions

- For a large H, curvature significantly effects the early universe SM instability
  - Running of couplings from H
  - A curvature mass $\propto \xi R\phi^2$ is always generated
- Stability during inflation and reheating constrains SM physics, namely for large H

$$\boxed{\xi \sim 1/6}$$

Thank You!