Results from the PVLAS attempt to measure vacuum magnetic birefringence and future outlook

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Summary

- The Superposition Principle
- Vacuum magnetic birefringence
- Present experimental method
- PVLAS results
- Experience from PVLAS
- VMB@CERN
- Possible problems

Classical electromagnetism in vacuum

Classical vacuum has no structure

In the absence of charges and currents

$$\boxed{\begin{aligned} \operatorname{div}\vec{\mathbf{D}} &= 0; \quad \operatorname{rot}\vec{\mathbf{E}} = -\frac{\partial\vec{\mathbf{B}}}{\partial t} \\ \operatorname{div}\vec{\mathbf{B}} &= 0; \quad \operatorname{rot}\vec{\mathbf{H}} = \frac{\partial\vec{\mathbf{D}}}{\partial t} \end{aligned}} \quad \Longleftrightarrow \quad \boxed{\mathcal{L}_{Class} = \frac{1}{2\mu_0}\left(\frac{\mathbf{E}^2}{c^2} - \mathbf{B}^2\right)}$$

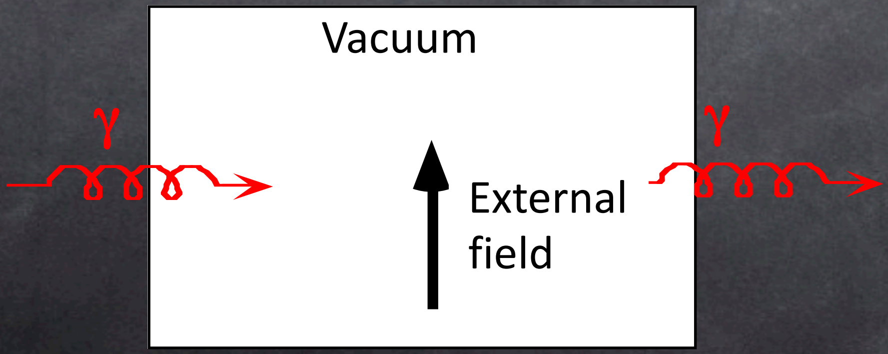
$$\boxed{\begin{aligned} \vec{\mathbf{D}} &= \frac{\partial\mathcal{L}_{Class}}{\partial\vec{\mathbf{E}}} \\ \vec{\mathbf{H}} &= -\frac{\partial\mathcal{L}_{Class}}{\partial\vec{\mathbf{B}}} \end{aligned}}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} \vec{\mathbf{D}} &= \epsilon_0\,\vec{\mathbf{E}} \\ \vec{\mathbf{B}} &= \mu_0\,\vec{\mathbf{H}} \end{aligned}}$$

Maxwell's equations are linear:
superposition principle holds

$$\boxed{c = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 299792458\ \mathrm{m/s}}$$



Vacuum: what is left when all that can be removed has been removed (J.C. Maxwell)

Inputs which changed the classical scenario

Three important discoveries changed the scenario:

- Einstein's mass-energy relation (1905)

$$\mathcal{E} = mc^2$$

- Heisenberg's Uncertainty Principle (1927)

$$\Delta\mathcal{E}\Delta t \geq \hbar/2$$

- Dirac's relativistic equation for the electron (1928) predicting negative energy states (discovery anti-matter 1932)

Vacuum can fluctuate

For example, virtual electron-positron pairs may 'exist' for a short time

First intuition of LbL interaction

O. Halpern, Phys. Rev. 44, pp 855, (1933)

Scattering Processes Produced by Electrons in Negative Energy States

Recent calculations[1] of the changes in the absorption-coefficient of hard gamma-rays due to the formation of electron-positron pairs have lent strong support to Dirac's picture of holes of negative energy. Still, the almost insurmountable difficulties which the infinite charge-density without field offers to our physical understanding make it desirable to seek further tests of the theory. Here purely

[1] J. R. Oppenheimer and M. S. Plesset, Phys. Rev. **44**, 53 (1933).

radiation phenomena are of particular interest inasmuch as they might serve in an attempt to formulate observed effects as consequences of hitherto unknown properties of corrected electromagnetic equations. We are seeking, then, scattering properties of the "vacuum."

Two possible types of phenomena must be considered separately in connection with the foregoing: (1) All incident quanta have the same direction of propagation; (2) The incident quanta have different directions of propagation. Since we are only interested in purely radiation phenomena the frequencies in the second case should lie below mc^2/h so that no permanent formation of electron-positron pairs can occur.

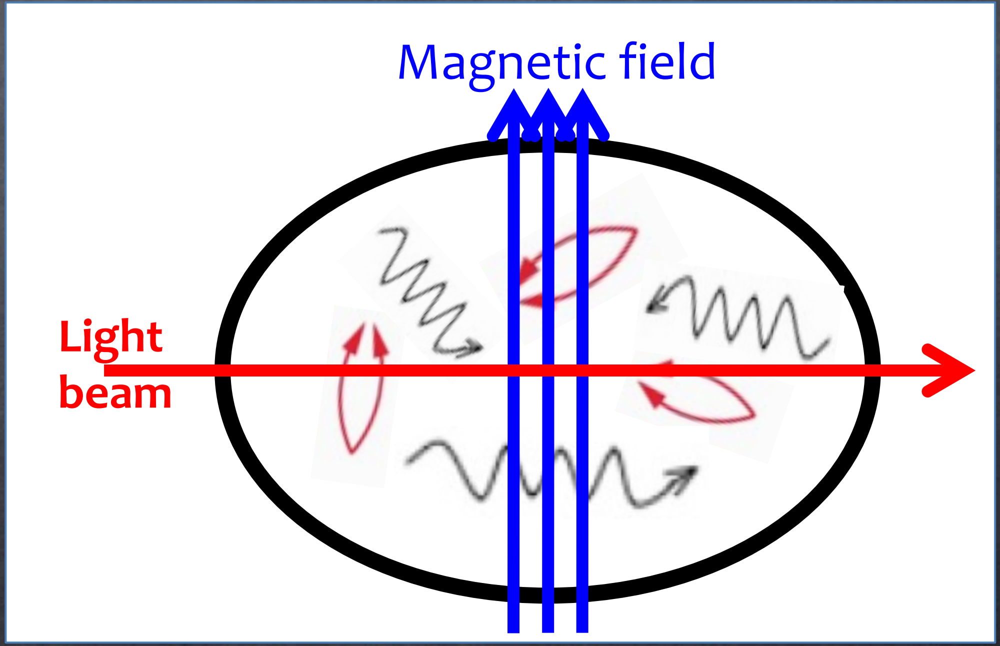
When all incident quanta have the same direction of propagation, the principle of conservation of momentum excludes all scattering processes other than those in the direction of the incident radiation. These scattering processes would be observable if accompanied by a change in frequency. In the language of Dirac's theory of radiation such splitting of the incident quantum occurs in processes of the following type: An electron in a negative energy state passes by absorption of the incident quantum into a state of positive energy; the electron then returns in several steps under emission of $h\nu$ in toto to its original state. At each step total momentum should be conserved.

Light propagation in an external field

- Experimental study of the propagation of light in vacuum in an external field
- General method
  - Perturb the vacuum with an external field
  - Probe the perturbed vacuum with polarized light

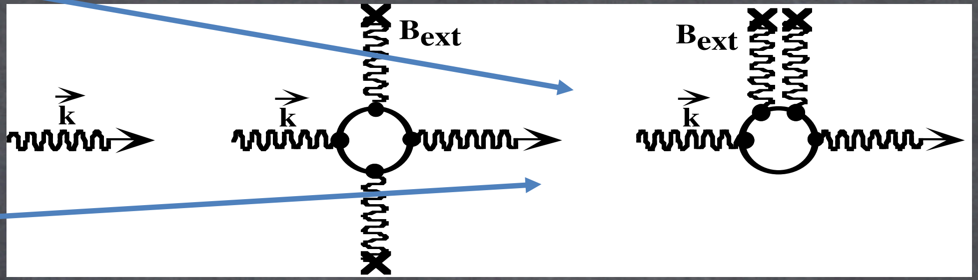
Measure variations of the index of refraction in vacuum due to the external magnetic field

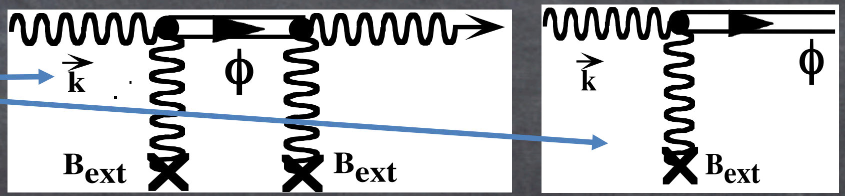
$$\tilde{n}_{\rm vac} = n_{\rm B} + i\kappa_{\rm B}$$



Physics reach

- Possible contributions to changes in $\tilde{n}_{\text{vac}}$
  - Vacuum magnetic birefringence
    - First order in light-by-light
  - Existence of:
    - Millicharged particles
    - Axion-like particles
    - …





  - Light-by-light interaction has been predicted but never yet measured directly at low energies.

Light propagation in an external field

In particular the interest is to study and hopefully measure

- LINEAR BIREFRINGENCE (polarisation dependent index of refraction)
- LINEAR DICHROISM (polarisation dependent index of absorption)

acquired by vacuum when subject to an external magnetic field

$$\Delta\tilde{n}_{\mathrm{vac}} = \Delta n_{\mathrm{B}} + i\Delta\kappa_{\mathrm{B}}$$

Predicted (QED)
Hypothetical (ALPs, MCPs)

Hypothetical (ALPs, MCPs)

birefringence

$$\Delta n_{\mathrm{B}} \propto B^2$$

$$n_{\mathrm{B},\parallel} \neq n_{\mathrm{B},\perp}$$

dichroism

$$\Delta\kappa_{\mathrm{B}} \propto B^2$$

$$\kappa_{\mathrm{B},\parallel} \neq \kappa_{\mathrm{B},\perp}$$

Linear birefringence

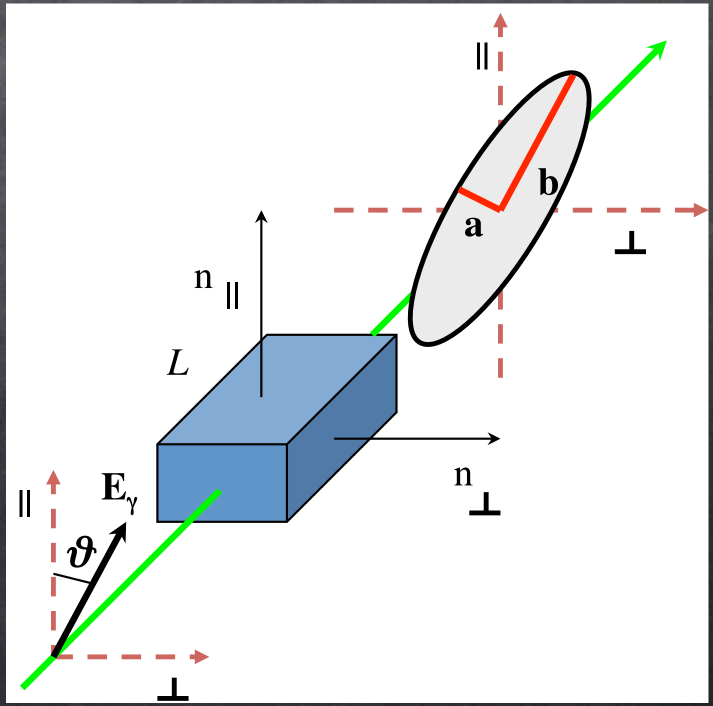
- A birefringent medium has $\Delta n = n_{||} - n_{\perp} \neq 0$
- A linearly polarized light beam propagating through a birefringent medium will acquire an ellipticity ψ

A linearly polarized light beam can be written as $\vec{E}_\gamma = E_\gamma e^{i\xi}\begin{pmatrix}1\\0\end{pmatrix}$

After propagating through the medium of length L the components of $\vec{E}_\gamma$ parallel and perpendicular to $\vec{B}$ will acquire a relative phase delay ϕ

$$\vec{E}_\gamma \approx E_\gamma e^{i\xi}\begin{pmatrix}1\\ i\frac{\phi}{2}\sin 2\vartheta\end{pmatrix}$$

Immaginary



Ellipticity $\psi = \frac{\phi}{2} = \frac{a}{b} \approx \frac{\pi \Delta n L}{\lambda}\sin 2\vartheta$

Linear dichroism

- A birefringent medium has $\Delta\kappa = \kappa_{||} - \kappa_{\perp} \neq 0$
- A linearly polarized light beam propagating through a dichroic medium will acquire a rotation ε

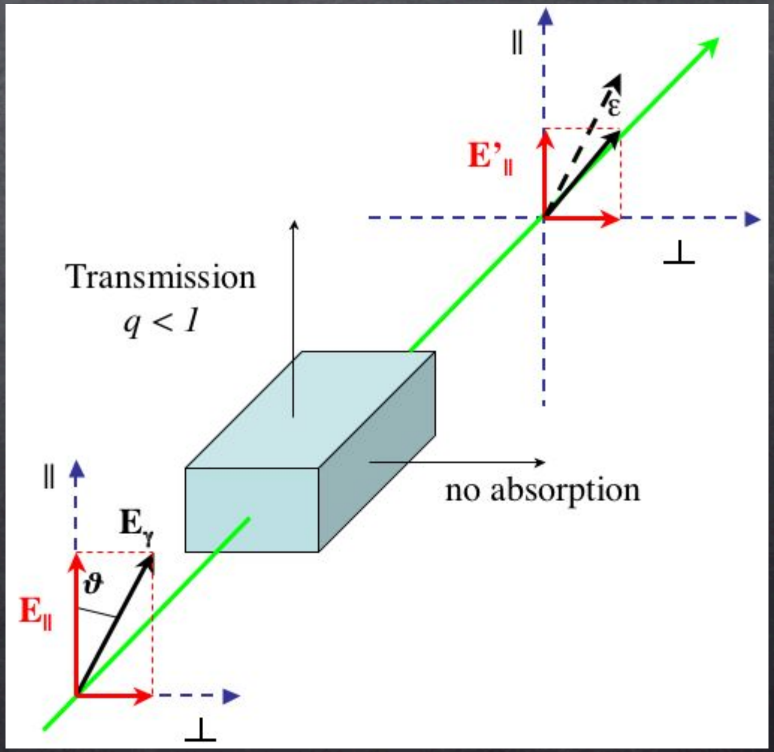
A linearly polarized light beam can be written as $\vec{E}_\gamma = E_\gamma e^{\imath\xi}\begin{pmatrix}1\\0\end{pmatrix}$

After propagating through the medium of length L the components of $\vec{E}_\gamma$ parallel and perpendicular to $\vec{B}$ will acquire a relative amplitude reduction ζ

$$\vec{E}_\gamma \approx E_\gamma e^{i\xi}\begin{pmatrix}1\\ \frac{\zeta}{2}\sin 2\vartheta\end{pmatrix}$$

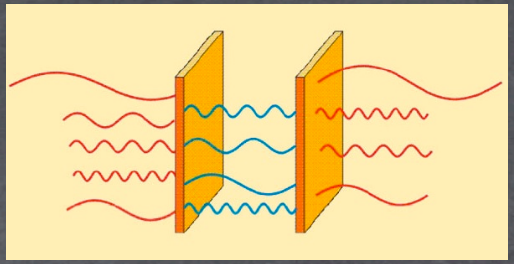
Real

Rotation $\varepsilon = \dfrac{\zeta}{2} \approx \dfrac{\pi\Delta\kappa L}{\lambda}\sin 2\vartheta$

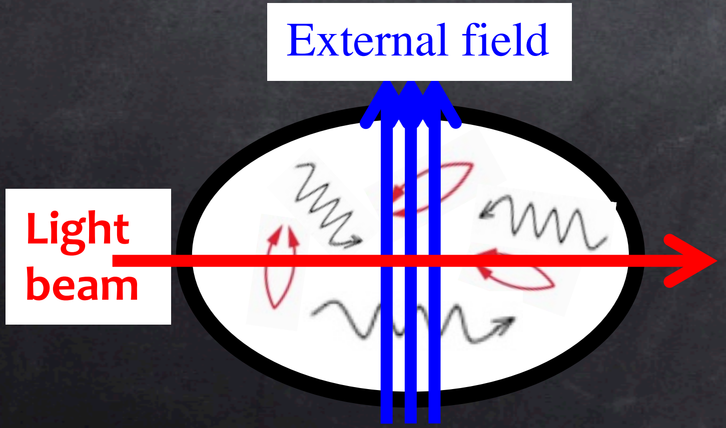


QED Tests

- Microscopic tests
  - QED tests in bound systems – Lamb shift, Delbrück scattering
  - QED tests with charged particles – (g-2)
- Macroscopic tests
  - Casimir effect (photon zero-point fluctuations)



- QED tests with only photons in the initial and final states are still missing. Vacuum magnetic birefringence is a macroscopic effect related to LbL interaction affecting directly the optical properties of vacuum and hence the speed of light.



Macroscopically observable (small) non-linear effects due to vacuum fluctuations have been predicted since 1935 but have never been directly observed yet.

H. Euler, B. Kockel (1935)

Euler and Kockel wrote an effective Lagrangian density describing electromagnetic interactions in the presence of the virtual electron-positron sea discussed a few years before by Dirac. This Lagrangian was then generalised by W. Heisenberg and later confirmed in the QED framework.

$$\boxed{\mathcal{L}_{\rm EK} = \frac{1}{2\mu_0}\left(\frac{E^2}{c^2} - B^2\right) + \frac{A_e}{\mu_0}\left[1\left(\frac{E^2}{c^2} - B^2\right)^2 + 7\left(\frac{\vec{E}}{c}\cdot\vec{B}\right)^2\right] + \ldots}$$

Coefficients determined by Euler and Kockel

$$A_e = \frac{2}{45\mu_0}\frac{\alpha^2 \lambda\!\!\!^{-}{}_e^3}{m_e c^2} = 1.32\times 10^{-24}\ \mathrm{T}^{-2}$$

H Euler and B Kochel, *Naturwissenschaften* **23**, 246 (1935)
W Heisenberg and H Euler, *Z. Phys.* **98**, 714 (1936)
H Euler, *Ann. Phys.* **26**, 398 (1936)
V Weisskopf, *Mat.-Fis. Med. Dan. Vidensk. Selsk.* **14**. 6 (1936)
J. Schwinger, *Phys. Rev.*, **82,** 664 (1951)

$$E \ll E_{\rm crit} = \frac{m_e^2 c^3}{e\hbar} = 1.3\times 10^{18}\ \mathrm{V/m}$$

$$B \ll B_{\rm crit} = \frac{E_{\rm crit}}{c} = 4.4\times 10^{9}\ \mathrm{T}$$

Leads to a non-linear behavior of electromagnetism in vacuum

Index of refraction - 1

Consider linearly polarized light propagating through a perpendicular magnetic field

By applying the constitutive relations to $\mathcal{L}_{\mathrm{EK}}$ one finds

$$\vec{D} = \frac{\partial \mathcal{L}_{EK}}{\partial \vec{E}} \qquad \vec{H} = -\frac{\partial \mathcal{L}_{EK}}{\partial \vec{B}}$$

$$\vec{D} = \epsilon_0 \mathbf{\vec{E}} + \epsilon_0 A_e \left[4\left(\frac{E^2}{c^2} - B^2\right)\vec{E} + 14\left(\vec{E}\cdot\vec{B}\right)\vec{B} \right]$$

$$\mu_0 \vec{H} = \vec{B} + A_e \left[4\left(\frac{E^2}{c^2} - B^2\right)\vec{B} - 14\left(\frac{\vec{E}}{c}\cdot\vec{B}\right)\frac{\vec{E}}{c} \right]$$

Light propagation can still be described by Maxwell's equations but in a medium. They no longer are linear due to E-K correction and the superposition principle no longer holds.

Index of refraction in the presence of a magnetic field

$$\begin{cases} \epsilon_\parallel = 1 + 10 A_e B_{\mathrm{ext}}^2 \\ \mu_\parallel = 1 + 4 A_e B_{\mathrm{ext}}^2 \\ n_\parallel = 1 + 7 A_e B_{\mathrm{ext}}^2 \end{cases} \qquad \begin{cases} \epsilon_\perp = 1 - 4 A_e B_{\mathrm{ext}}^2 \\ \mu_\perp = 1 + 12 A_e B_{\mathrm{ext}}^2 \\ n_\perp = 1 + 4 A_e B_{\mathrm{ext}}^2 \end{cases}$$

$$\boxed{\Delta n = 3 A_e B_{\mathrm{ext}}^2}$$

Index of refraction - 2

$\boxed{\begin{array}{l} n_{\parallel,\perp} \neq 1 \\ n_{\parallel} - n_{\perp} \neq 0 \end{array}}$ ⇒ $v < c$ anisotropy ⇒ A_e can be determined directly by measuring the magnetic birefringence of vacuum.

$$\Delta n_{(\alpha^2)} = 3A_e B_{\text{ext}}^2$$

$$\Delta n_{(\alpha^3)} = 3A_e B_{\text{ext}}^2 \left(1 + \frac{25}{4\pi}\alpha\right)$$

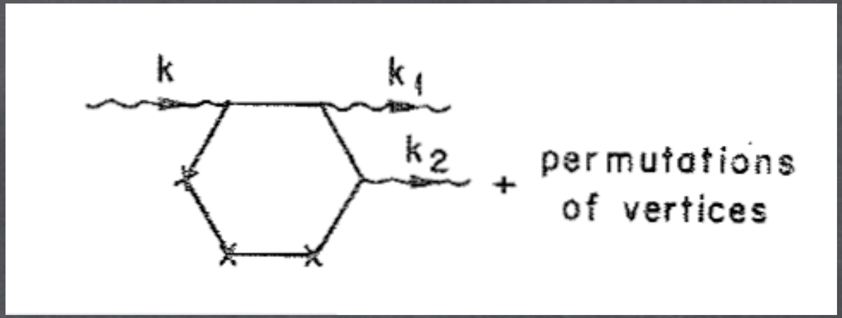
Radiative correction contribution ≈ 1.45%

Minute effect $\boxed{\Delta n^{(\text{QED})} = 2.5 \times 10^{-23} \ @ \ 2.5 \text{ T}}$

Index of refraction - 3

S. Adler (1971) calculated the absorption due to QED which is connected to the phenomenon known as photon splitting

Lowest order low energy photon splitting diagram



$$\mu_{\binom{\perp}{\parallel}} = \frac{4\pi}{\lambda}\kappa_{\binom{\perp}{\parallel}} = \begin{pmatrix}0.51\\0.24\end{pmatrix}\left[\frac{\hbar\omega}{m_e c^2}\right]^5\left[\frac{B\sin\vartheta}{B_{\rm cr}}\right]^6 \ \mathrm{cm}^{-1}$$

Summing up

Expected value for $n_{\rm vac}$

$$\tilde{n}_{\rm vac} = n_{\rm B} + i\kappa_{\rm B}$$

$$\boxed{n_{\binom{\perp}{\parallel}} = 1 + \begin{pmatrix}4\\7\end{pmatrix}\times \underline{1.32\cdot 10^{-24}}\left(\frac{B}{1\,\mathrm{T}}\right)^2 + i\begin{pmatrix}0.51\\0.24\end{pmatrix}\times\underline{4\cdot 10^{-91}}\left(\frac{\lambda}{1\,\mu\mathrm{m}}\right)\left(\frac{B}{1\,\mathrm{T}}\right)^6\left(\frac{\hbar\omega}{1\,\mathrm{eV}}\right)^6}$$

$$A_e = \frac{2}{45\mu_0}\frac{\alpha^2 \lambda\!\!\!^{-}{}_e^3}{m_e c^2}$$

Unmeasurably small

Axion-Like Particles (ALPs)

Extra terms can be added to the effective Lagrangian to include contributions from hypothetical neutral light particles weakly interacting with two photons

g_a, g_s coupling constants

pseudoscalar case

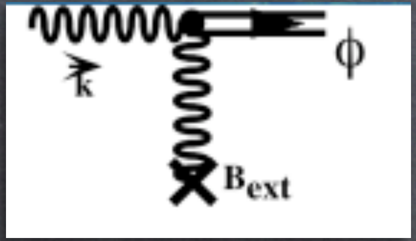
$$\mathcal{L}_a = g_a \phi_a \vec{E}_\gamma \cdot \vec{B}_{\text{Ext}}$$

scalar case

$$\mathcal{L}_s = g_s \phi_s \vec{B}_\gamma \cdot \vec{B}_{\text{Ext}}$$

both interactions are polarisation dependent

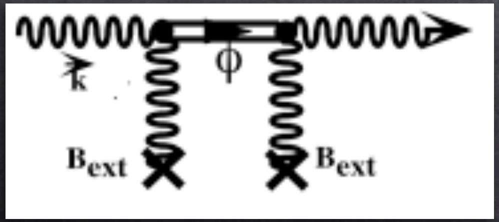
Dichroism



$$|\Delta\kappa_{\text{B}}^{(\text{ALP})}| = \kappa_{\parallel}^{a} = \kappa_{\perp}^{s} = \frac{2}{\omega L}\left(\frac{g_{a,s} B_{\text{ext}} L}{4}\right)^2 \left(\frac{\sin x}{x}\right)^2$$

$$x = \frac{L m_{a,s}^2}{4\omega}$$

Birefringence

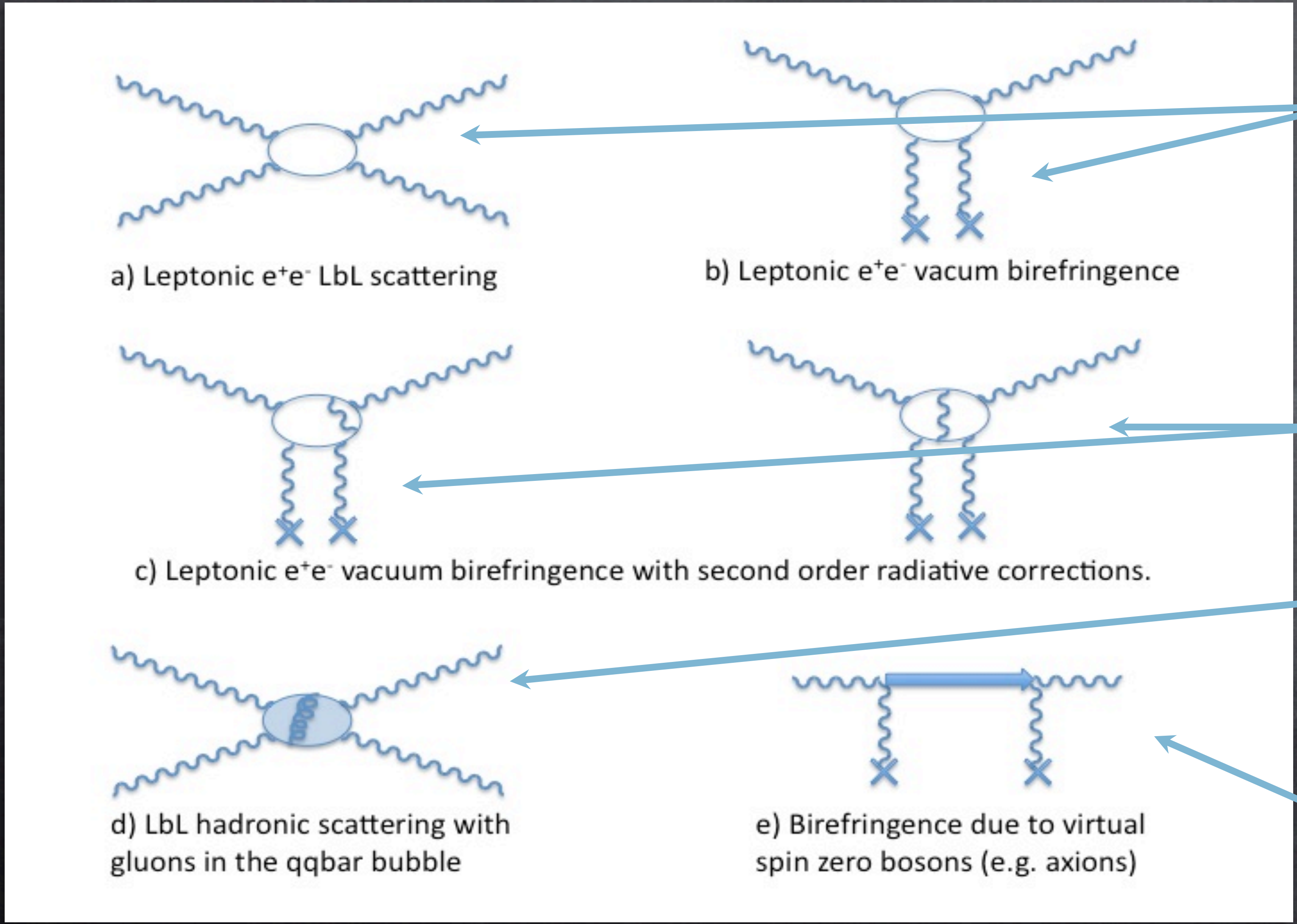


$$|\Delta n_{\text{B}}^{(\text{ALP})}| = n_{\parallel}^{a} - 1 = n_{\perp}^{s} - 1 = \frac{g_{a,s}^2 B_{\text{ext}}^2}{2 m_{a,s}^2}\left(1 - \frac{\sin 2x}{2x}\right)$$

Maiani L, Petronzio R, Zavattini E, Phys. Lett B 173, 359 (1986)
Raffelt G and Stodolsky L Phys. Rev. D 37, 1237 (1988)

$$1\,\text{T} = \sqrt{\frac{\hbar^3 c^3}{e^4 \mu_0}} = 195\,\text{eV}^2 \qquad 1\,\text{m} = \frac{e}{\hbar c} = 5.06 \times 10^6\,\text{eV}^{-1}$$

Summary of possible 4 photon processes



Described by the Euler-Kockel-Heisenberg Lagrangian. Must be there.

Also includes MCPs

Corrections 1.45%

Hadronic contribution. Difficult to extract from indirect measurements. g-2 open problem.

Contribution from hypothetical new particles coupling to two photons.

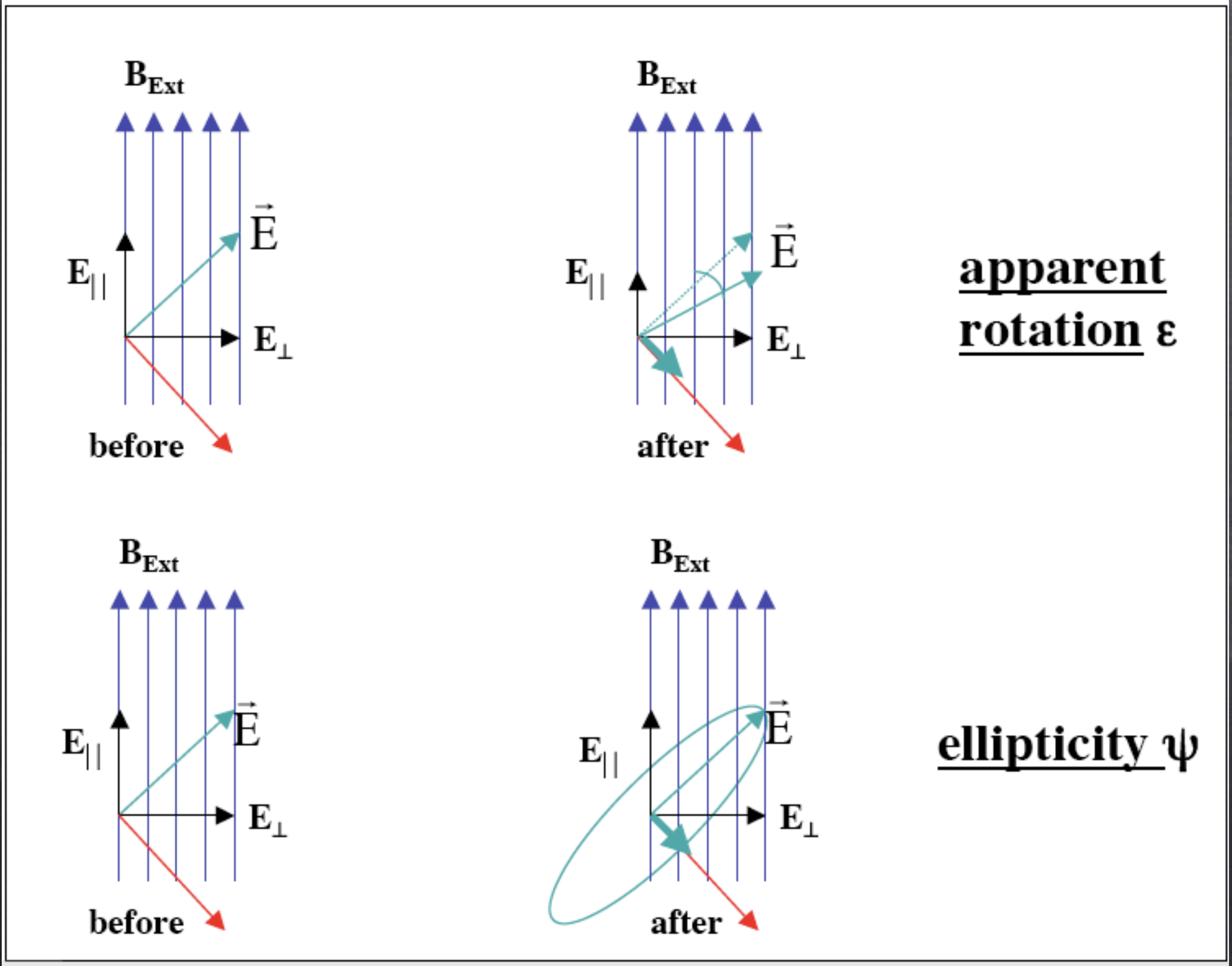
Summing up ...

Dichroism $\Delta\kappa$

- (Photon splitting)
- ALPs, MCPs

Birefringence Δn

- QED
- ALPs, MCPs



Both Δn and $\Delta\kappa$ are defined with sign

Recent results on Light-by-Light interaction

- ATLAS has indirectly observed γ - γ interactions at high energies in lead-lead peripheral interactions at LHC.
- These results received lots of interest.

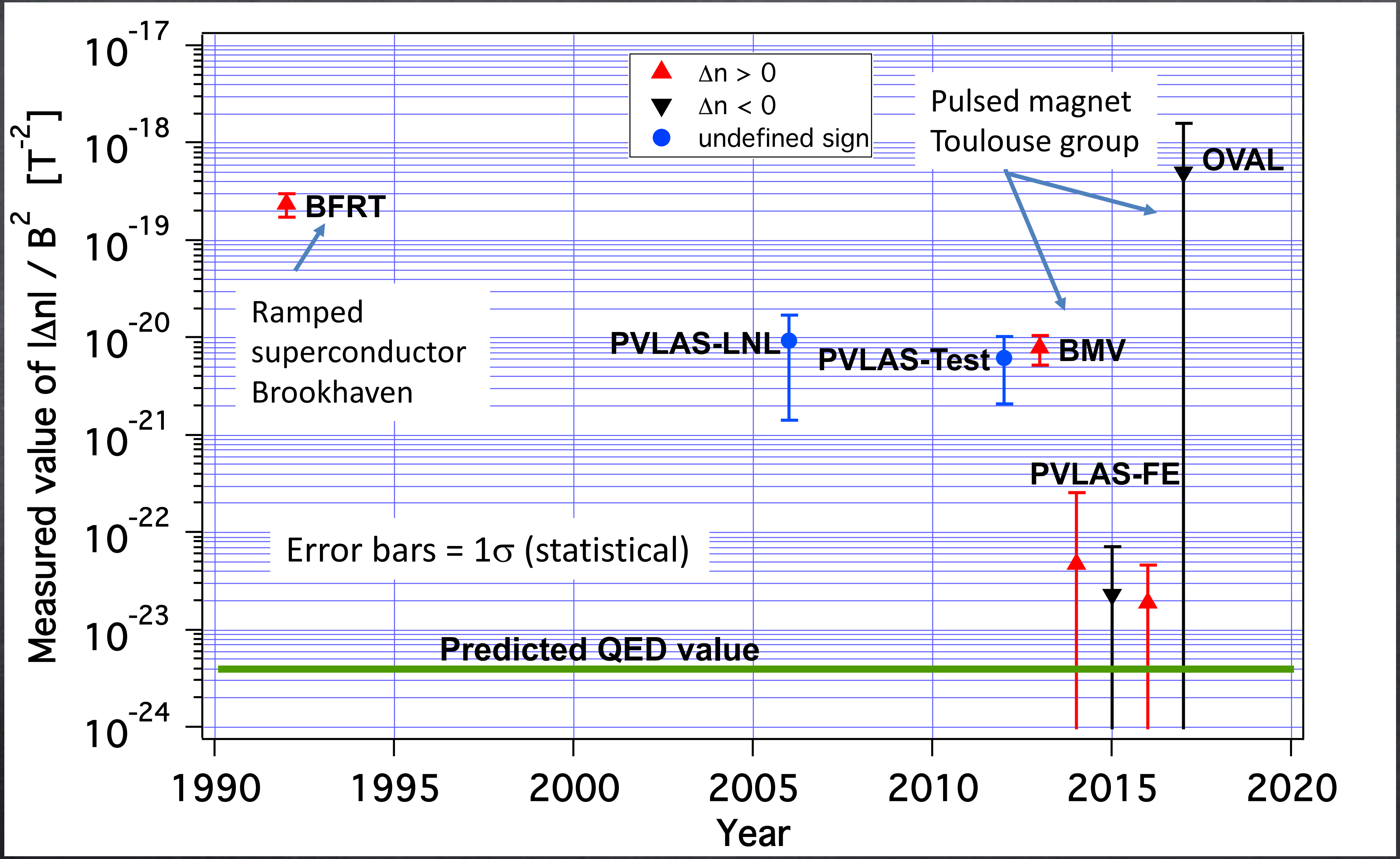
But

- In scattering processes there is no modification of the optical properties of vacuum, i.e. no modification of the vacuum electromagnetic constants c, ε_0 and μ_0.
- Mignani et al. have indirectly inferred evidence of vacuum magnetic birefringence from optical polarimetry of an isolated neutron star.
- Low energy macroscopic non-linear vacuum effects remain a topic of great interest still lacking a direct laboratory confirmation.

- Mignani et al., Monthly Notices of the Royal Astronomical Society, Volume 465, Issue 1, 11 February 2017, Pages 492–500
- ATLAS collaboration, Nature Physics 13, 852–858 (2017); ATLAS Collaboration, Phys. Rev. Lett. **123**, 052001 (2019)

Birefringence evolution of results

- 2016 PVLAS result corresponds to an integration $T \approx 5 \cdot 10^6$ s.



Experimental method

Iacopini and Zavattini proposal (1979)

Volume 85B, number 1 PHYSICS LETTERS 30 July 1979

EXPERIMENTAL METHOD TO DETECT THE VACUUM BIREFRINGENCE INDUCED BY A MAGNETIC FIELD

E. IACOPINI and E. ZAVATTINI
CERN, Geneva, Switzerland

Received 28 May 1979

In this letter a method of measuring the birefringence induced in vacuum by a magnetic field is described: this effect is evaluated using the non-linear Euler–Heisenberg–Weisskopf lagrangian. The optical apparatus discussed here may detect an induced ellipticity on a laser beam down to 10^{-11}.

- First proposal to use a polarimeter to measure vacuum magnetic birefringence
- The basic measurement principle is still the same today.

Key ingredients to measuring $\Delta n = 3A_e B_{\text{ext}}^2$

- High magnetic field
  modulated in time in amplitude or direction
- Long effective optical path
  very-high finesse Fabry-Perot resonator: $L_{\text{eff}} = NL; \quad N = 2\mathcal{F}/\pi$
- Ellipsometer with heterodyne detection for best sensitivity
  signal modulation beats with a carrier signal

$$\Psi = \frac{\pi L_{\text{eff}}}{\lambda}\Delta n \sin 2\vartheta = \frac{2\mathcal{F}}{\pi}\psi \sin 2\vartheta$$

Optical path multiplier

Single pass ellipticity

The magnetic field source

Magnetic field must be normal to light propagation: dipole magnet.
In order to employ heterodyne detection the magnetic field must vary in time.

- Superconducting magnet:
  - Allows for high field (up to 10 T)
  - Needs cryogenics – low duty cycle
  - Modulation by current ramping (low frequency ≈ mHz) or magnet rotation (Hz)
  - Cumbersome system
- Permanent magnet:
  - Smaller field (up to 3 T)
  - Field always present –> duty cycle = 1
  - Modulation only by rotation
  - High frequency (5-20 Hz)
- Normal conducting magnet:
  - Much smaller field for continuous operation
  - Very high field for millisecond pulsed operation (> 20 T)
  - High modulation frequency
  - Low duty cycle -> critical

All have been tried

Experimental evolution of PVLAS

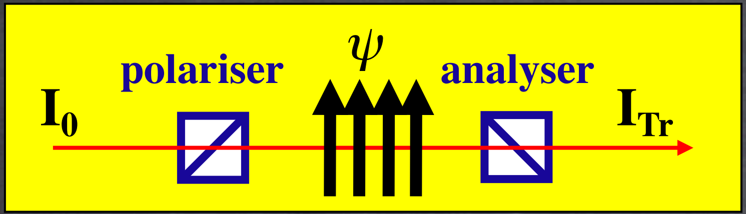
- CERN precursor experiment (1979 - 1983)
  - Superconducting magnet ~ 1 T
  - Delay line optical cavity, length amplification ~ 100
  - Sensitivity not sufficient for vacuum measurement
- Brookhaven experiment (1986 - 1990)
  - 2 superconducting magnets, 8.8 m, B_0 = 3.25 T, ΔB = 0.62 T ramped @ 30 mHz
  - Delay line optical cavity, length amplification ~ 500
  - Improved limits on axion like particles
- PVLAS @ Legnaro (Padova, Italy) (1992 - 2008)
  - Superconducting magnet, 1 meter, 5.5 T, rotating cryostat + magnet @ 0.4 Hz
  - Ellipsometer with 6.4 m Fabry - Perot cavity
  - Length amplification ~ 70 000
- PVLAS @ Ferrara Test (Ferrara, Italy) (2006- 2013)
  - Permanent magnets, 2 X 0.2 m, 2.3 T, rotating @ 3 Hz
  - Ellipsometer with 1.4 m Fabry - Perot cavity
  - Length amplification ~ 250 000
- PVLAS @ Ferrara (Ferrara, Italy) (2011- 2019)
  - Permanent magnets, 2 X 0.9 m, 2.5 T, rotating @ 20 Hz
  - Ellipsometer with 3.3 m Fabry - Perot cavity
  - Length amplification ~ 430 000

PVLAS @ FERRARA

The PVLAS (2009 - 2018) Collaboration:

Ferrara:	Aldo Ejlli (now Cardiff), Ugo Gastaldi, Giuseppe Messineo, Guido Zavattini
Legnaro:	Ruggero Pengo, Giuseppe Ruoso
Trieste:	Federico Della Valle (now Siena), Edoardo Milotti

Heterodyne detection - rotating magnet

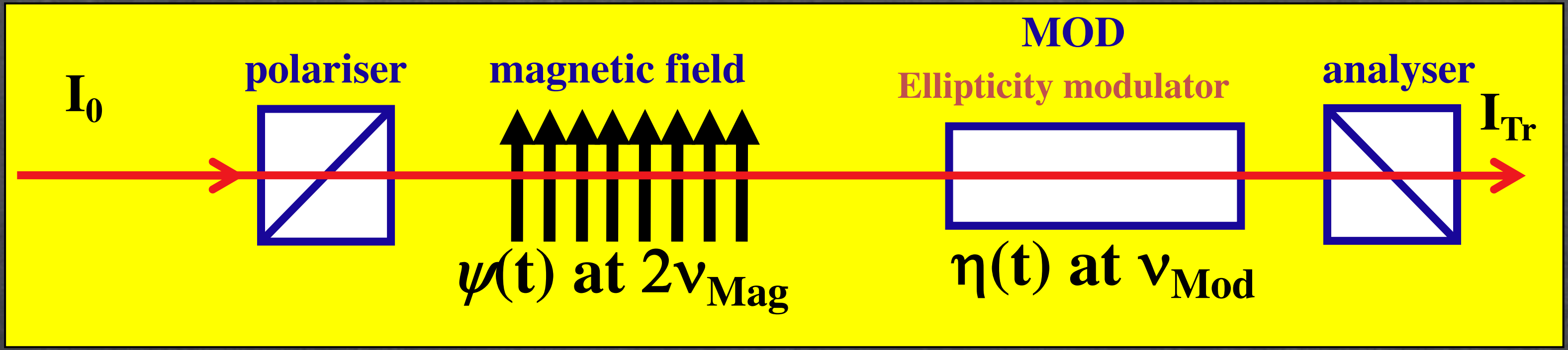


Static detection excluded (single pass): $\psi_{QED} \approx 10^{-16}$

$$I_{\text{out}} = I_0 \left| i\psi + \varepsilon \right|^2 = I_0 \left(\psi^2 + \varepsilon^2 \right)$$

Furthermore, rotations and ellipticities do not mix

Add a known time varying ellipticity $\eta(t)$ to $\psi \sin 2\vartheta(t)$. With $\eta, |\psi| << 1$, these add algebraically.



ϑ

$$I_{\text{out}} = I_0 |i\psi \sin 2\vartheta(t) + i\eta(t)|^2 = I_0 \left[\eta^2(t) + 2\eta(t)\psi \sin 2\vartheta(t) + \dots \right]$$

The intensity I_{out} is now linear in the ellipticity ψ.

Heterodyne detection - rotating magnet

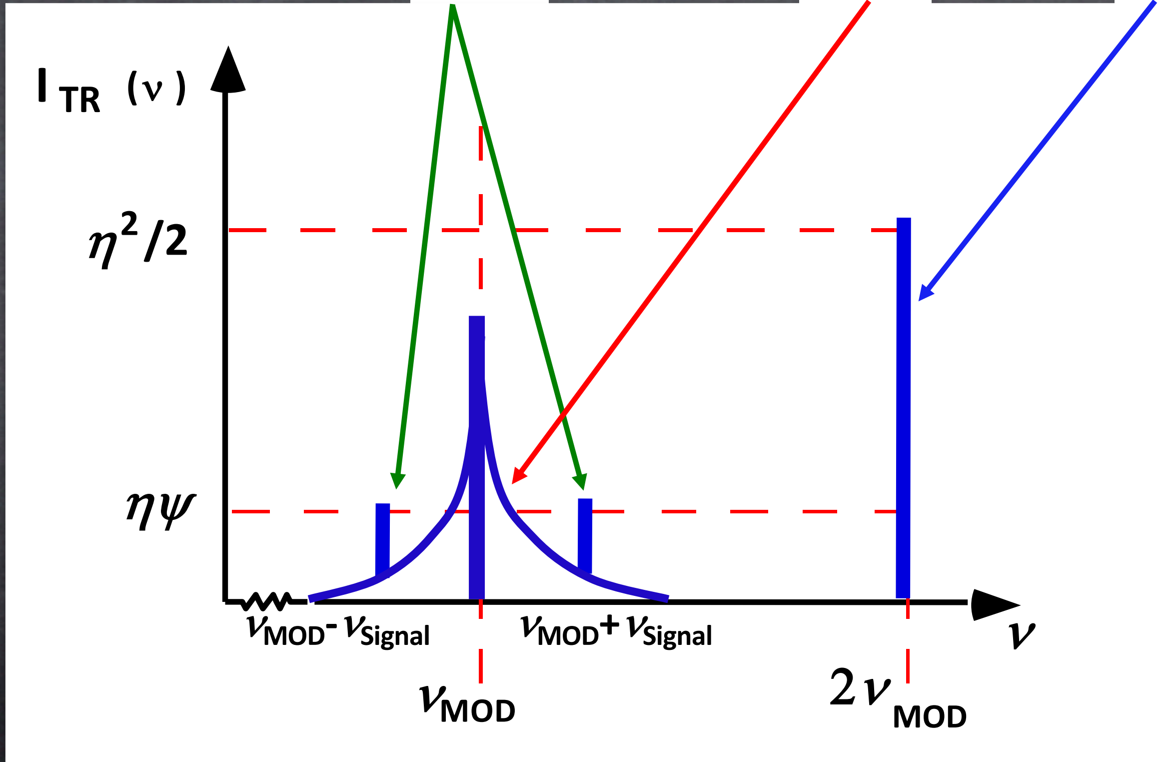
- In practice slowly varying spurious ellipticities $\alpha(t)$ are also present and the polarisers have an extinction factor σ^2.
- $\psi \sin 2\vartheta(t)$ is modulated in time by rotating the magnetic field. In PVLAS we have permanent magnets.
- By modulating both $\eta(t)$ and $\vartheta(t)$ the double product leads to frequency sidebands around the $\eta(t)$ carrier frequency.
- The $\eta^2(t)$ term results at twice the carrier frequency and is used to measure η_0 directly.

$$\boxed{I_{\text{out}} = I_0 \left[\sigma^2 + \eta(t)^2 + + \alpha(t)^2 + 2\eta(t)\underbrace{\psi \sin 2\vartheta(t)}_{\psi(t)} + 2\eta(t)\alpha(t) \ldots\right]}$$

Fourier spectrum of I_{out}

With $\eta(t)$ and $\psi(t)$ sinusoidal functions

$$I_{\text{out}} = I_0 \left[\sigma^2 + \underbrace{2\eta(t)\psi(t)}_{\text{signal}} + \underbrace{2\eta(t)\alpha(t)}_{\text{noise}} + \underbrace{\eta(t)^2}_{\text{normalization}} + \alpha(t)^2 + \ldots\right]$$

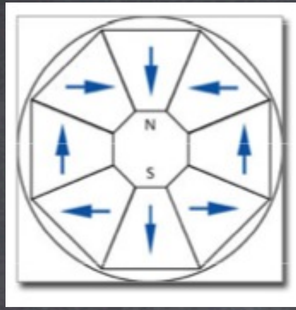


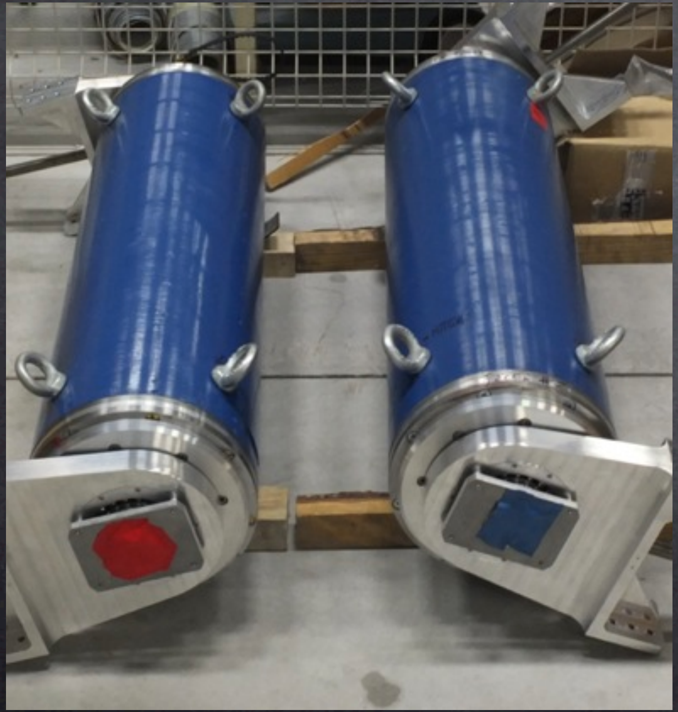
$$\nu_{\text{Signal}} = 2\nu_{\text{Mag}}$$

- I_{out} is then demodulated at ν_{mod} obtaining a low frequency single sided spectrum.
- Demodulating at $2\nu_{mod}$ allows the determination of the modulation amplitude η_0.

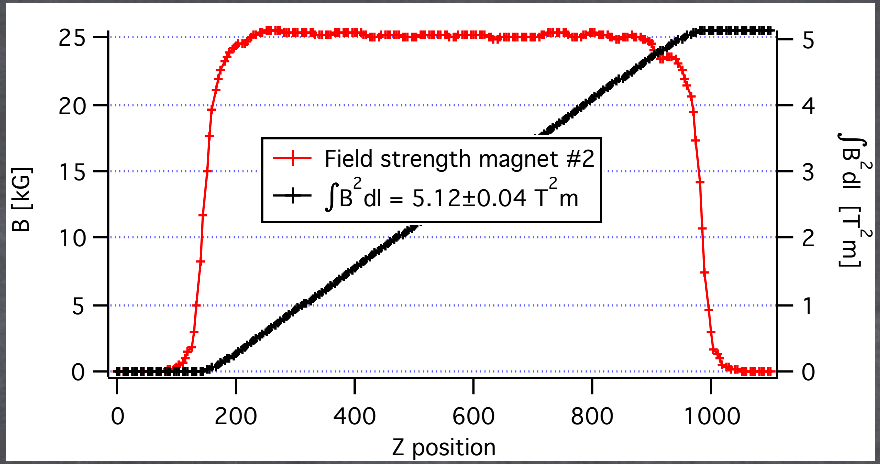
Permanent rotating magnets

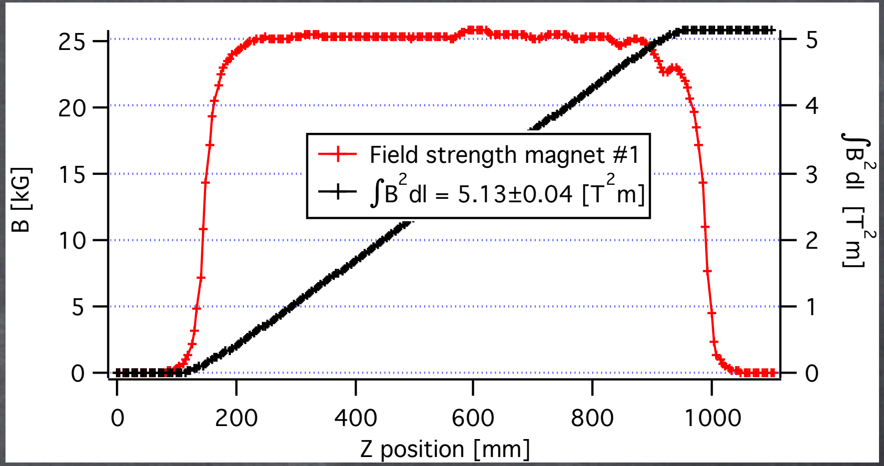
Halbach configuration





Total field integral = (10.25±0.06) T^2m





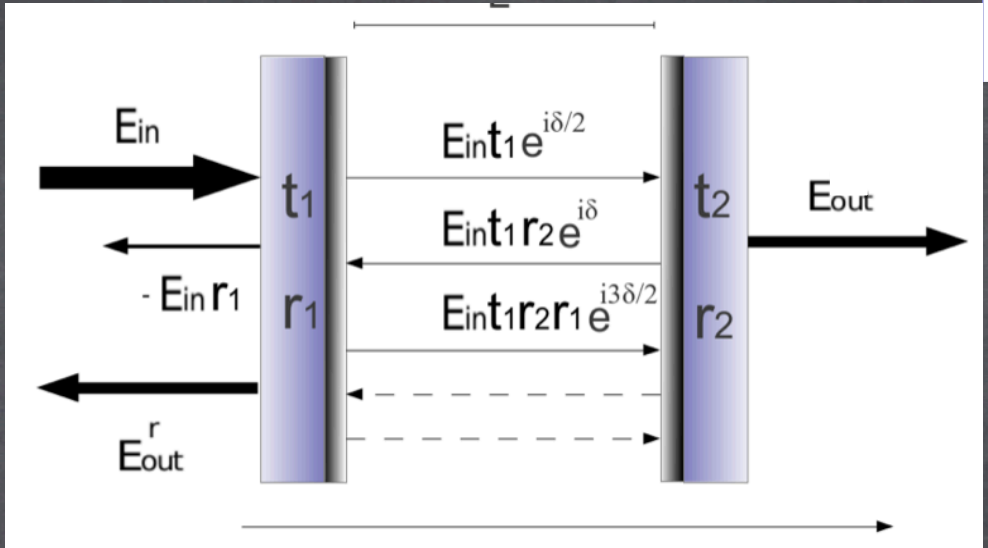
External diameter ϕ_{ext}	280 mm
Internal free bore diameter ϕ_{in}	20 mm
Overall length	938 mm
Magnetic field in the center of the bore	2.5 T
Magnetic stray field on axis 20 cm outside	10^{-4} T
Mass	450 kg
Magnetic material	$Nd_2Fe_{14}B$
Longitudinal sectors	12
Number of wedges per sector	16

Magnets have built in **magnetic shielding**
Stray field below 1 Gauss on side

Optical path multiplier

- The ellipticity induced by a birefringence is proportional to the path length in the magnetic region
- A Fabry-Perot interferometer is used to increase the path length by a factor up to 430’000. A magnet 1 meter long becomes equivalent to 430 km!
- Very high reflectivity mirrors with very low losses (≈ ppm) are used
- A critical standing wave condition is maintained with a feedback system applied to the laser

Fabry-Perot - 1



t and r are the reflection coefficients of the electric field

Let us assume $t_1 = t_2$ and $r_1 = r_2$.

Ideally $t^2 + r^2 = 1$

The roundtrip phase of a wave is $\delta = \dfrac{4\pi n L}{\lambda}$

The electric field at the output of the system will be

$$E_{\text{out}} = E_{\text{in}} t^2 e^{i\frac{\delta}{2}} \sum_{n=0}^{\infty} r^{2n} e^{ni\delta} = E_{\text{in}} t^2 \frac{e^{i\frac{\delta}{2}}}{1 - r^2 e^{i\delta}}$$

Resonance condition: $\delta = 2m\pi$ for which $E_{\text{out}} = \pm E_{\text{in}}$

Near resonance: $\delta' = 2m\pi + 2\phi$ (for example if L and λ are fixed but n changes)

Phase shift of E_{out} will be $\boxed{\phi_{\text{out}} = \dfrac{1+r^2}{1-r^2}\phi}$ with r^2 close to 1.

Fabry-Perot - 2

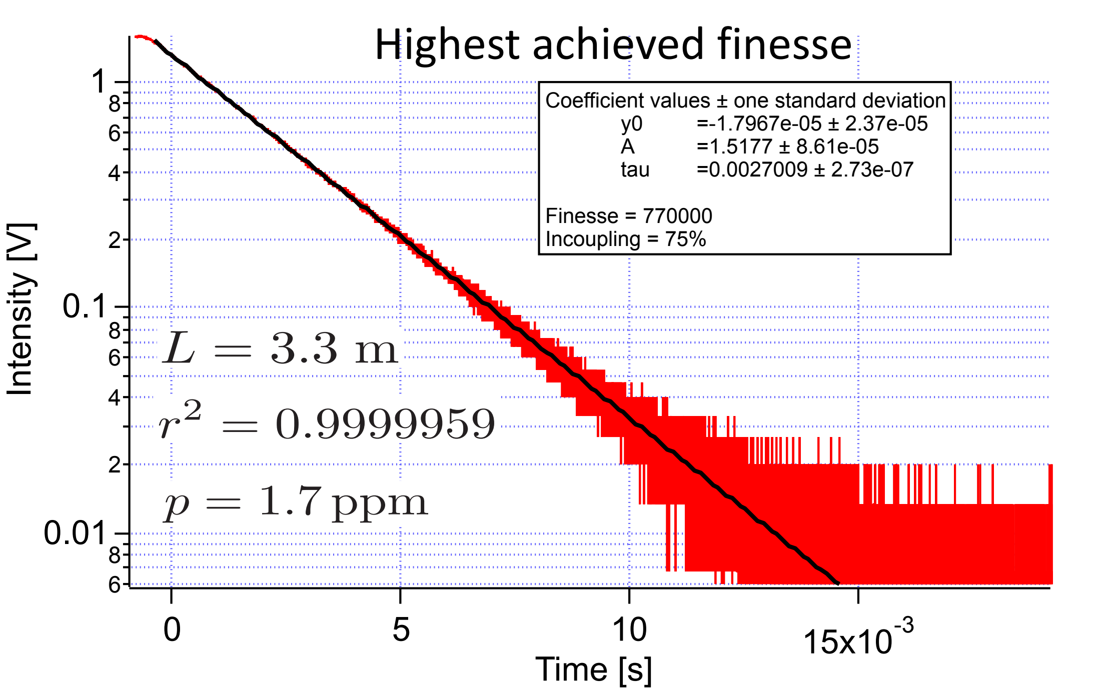
A relative phase shift between the parallel and perpendicular components of $\vec{E}_\gamma$ with respect to $\vec{B}$ will therefore be amplified and hence the ellipticity too. (With loses: $t^2 + r^2 + p = 1$)

$$\Rightarrow \quad E_\perp = E_0 \frac{t^2}{t^2+p}\left[i\alpha(t) + i\eta(t) + i\frac{1+r^2}{1-r^2}\psi \sin 2\vartheta(t)\right]$$

where $\dfrac{1+r^2}{1-r^2} = N = \dfrac{2\mathcal{F}}{\pi}$ and $\mathcal{F}$ is the finesse of the cavity.

$\mathcal{F}$ can be directly determined by measuring the light decay curve.

$$\mathcal{F} = \frac{\pi}{1-r^2} = \frac{\pi c\tau}{L} = 770000$$



F. Della Valle et al., Optics Express Vol. 22, pp. 11570-11577 (2014)

Numerical values for PVLAS @ Ferrara

Main interest of PVLAS is the Euler-Heisenberg birefringence

- $B = 2.5$ T
- $L_{\rm eff} = 2NL_B \approx 750$ km

$$\Delta n = 3A_e B^2 = 2.5\cdot 10^{-23} \quad \Rightarrow \quad \Psi = 5\cdot 10^{-11}$$

The necessary sensitivity to reach SNR = 1 in 10^6 s (12 days) is

$$S_\Psi < 5\cdot 10^{-8}\ 1/\sqrt{\rm Hz}$$

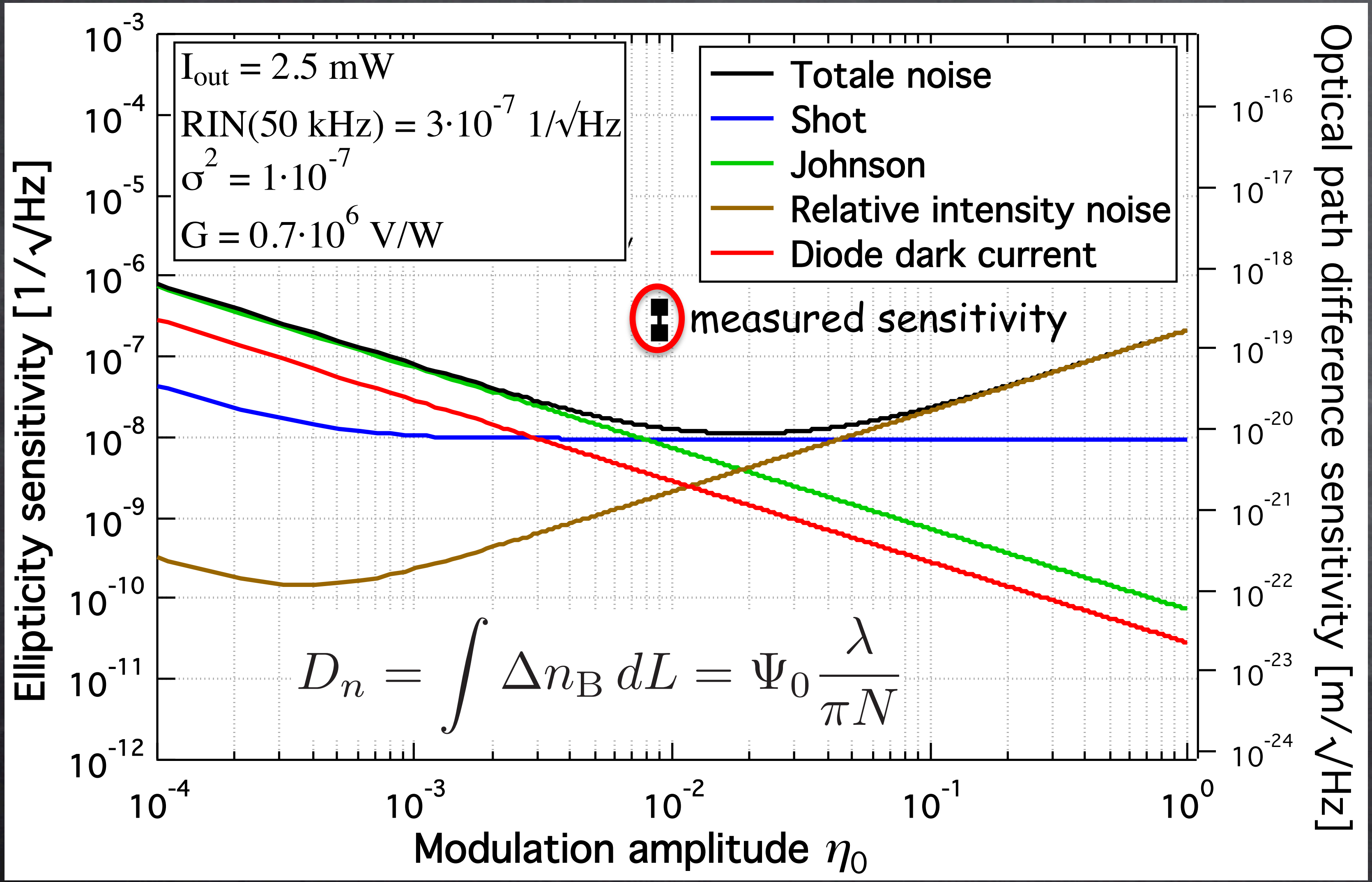
Shot noise limit = $\sqrt{\dfrac{e}{I_0 q}} = 10^{-8}\ \dfrac{1}{\sqrt{\rm Hz}}$ for $I_0 = 2.5$ mW

(I_0 = output intensity reaching the analyzer, q = 0.7 A/W)

In principle the effect should be detectable in 11 hrs

Measured sensitivity

Sensitivity as a function of modulation amplitude. Finesse = 700’000



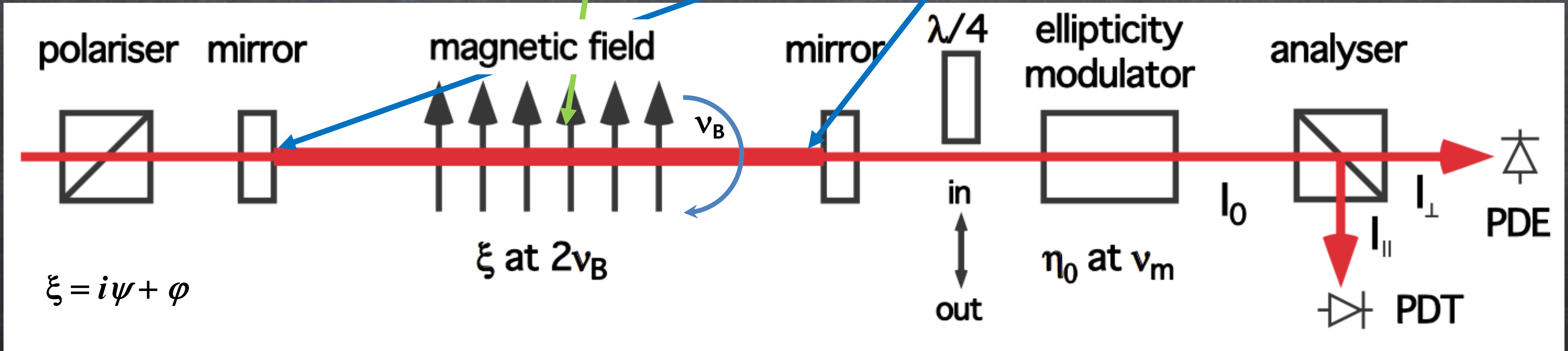
- Shot noise is reached without the Fabry-Perot cavity
- The extra noise is due to the presence of the cavity

Summarizing PVLAS scheme

$$I_{\text{out}} \simeq I_0 \left\{ \eta^2(t) + 2\eta(t) N \psi \sin 2\vartheta(t) + 2\eta(t) N \alpha(t) + \varphi^2(t) + \dots \right\}$$

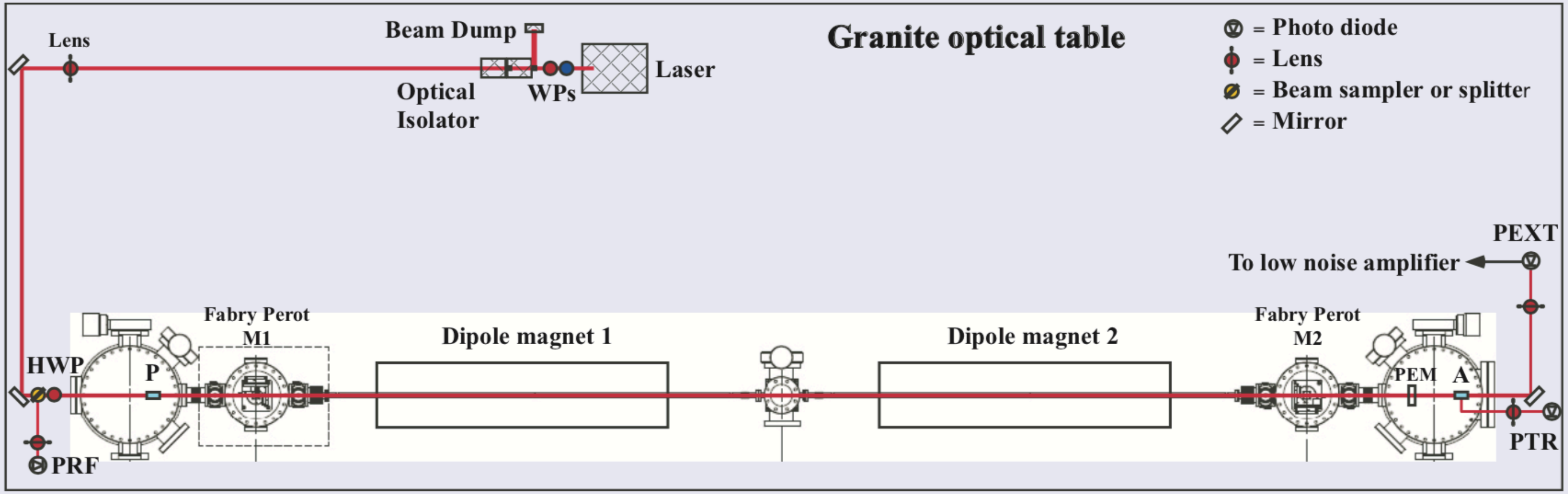
modulator signal noise rotation

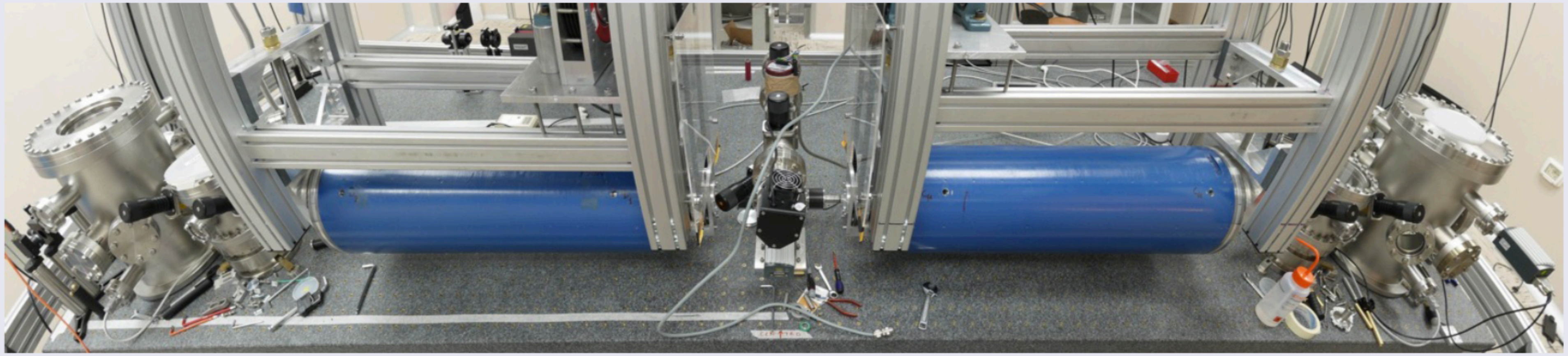
F. Della Valle et al. Eur. Phys. J. C (2016) 76:24



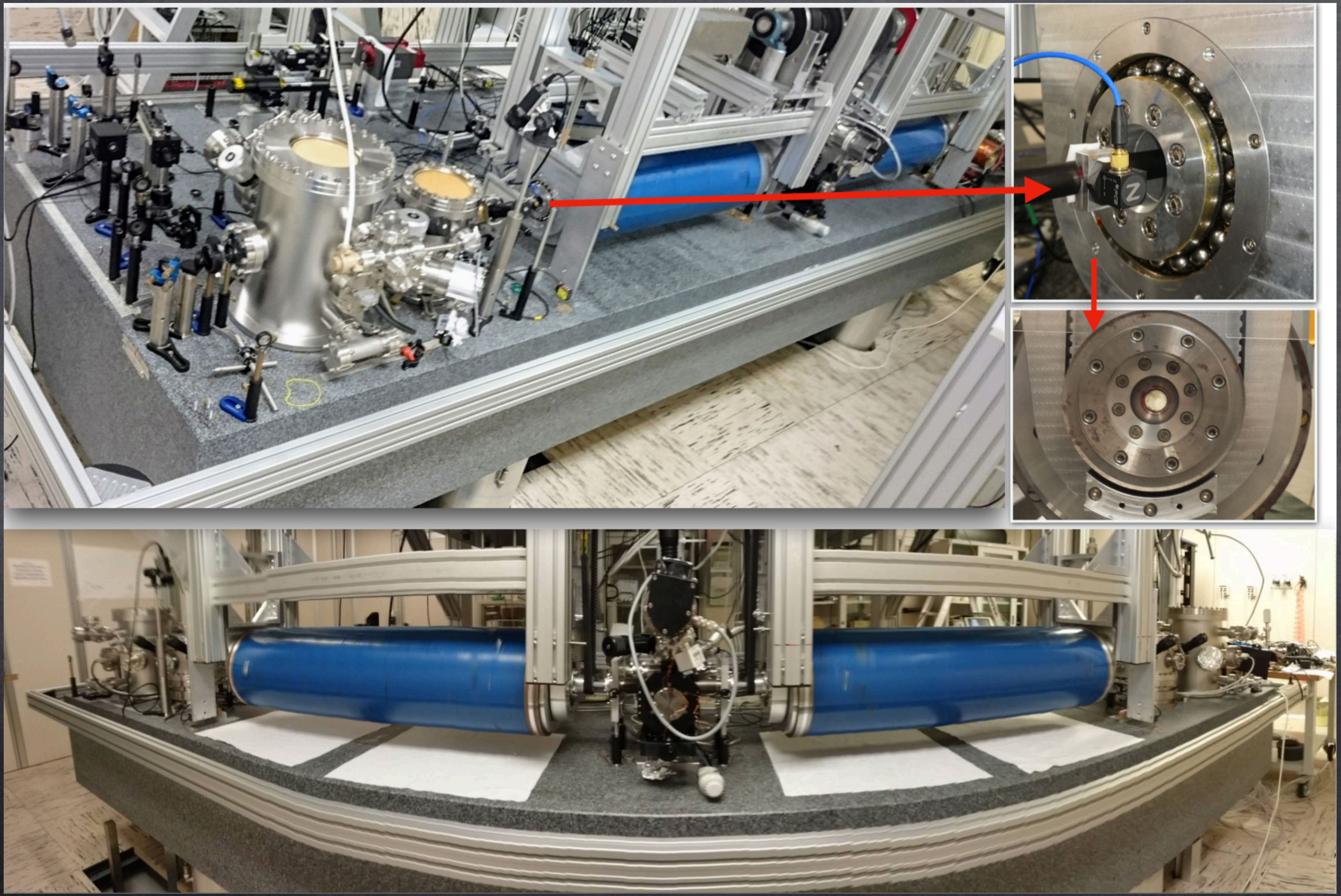
The extra noise comes from the mirrors

The mounted PVLAS apparatus

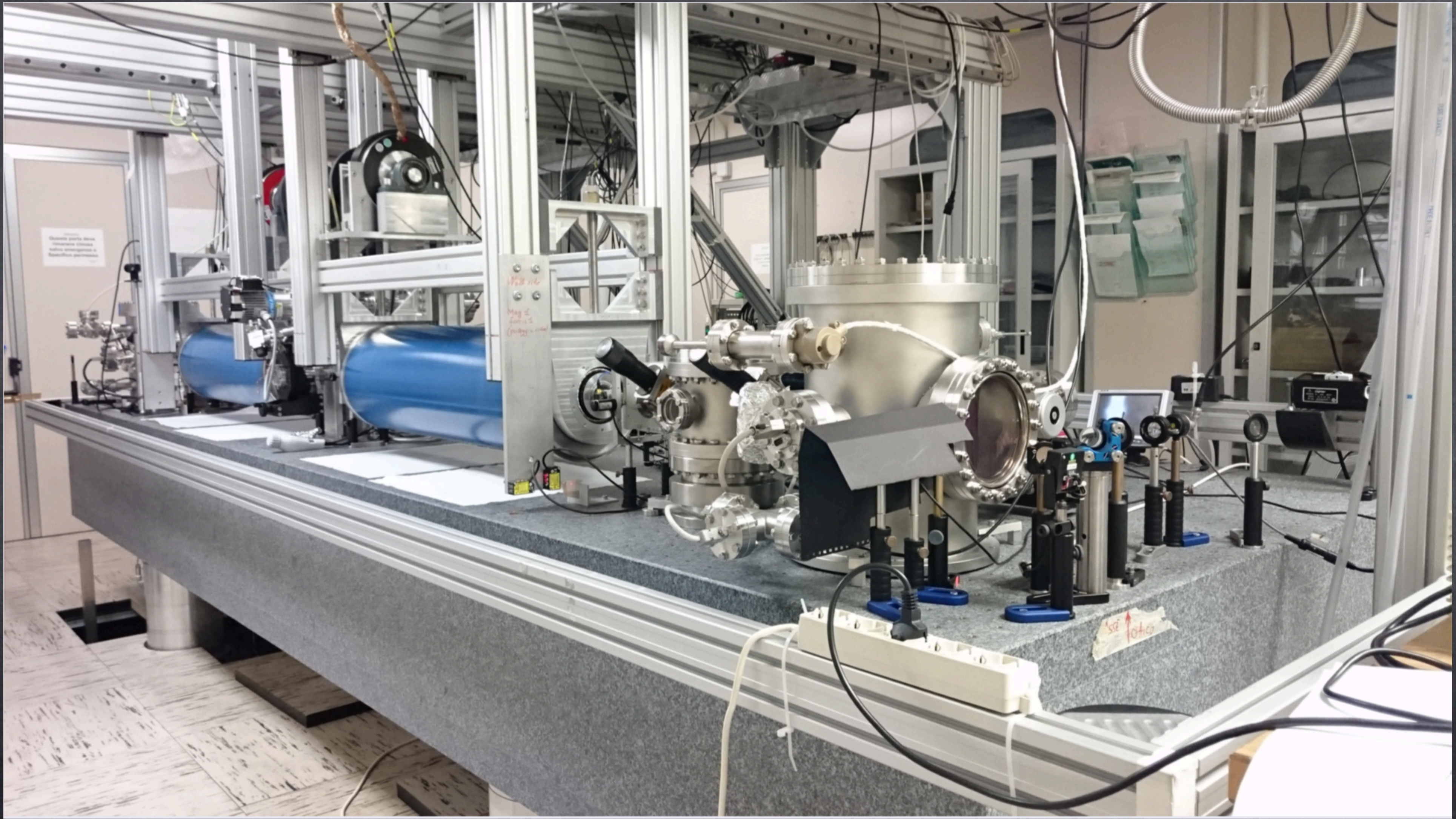




PVLAS input end

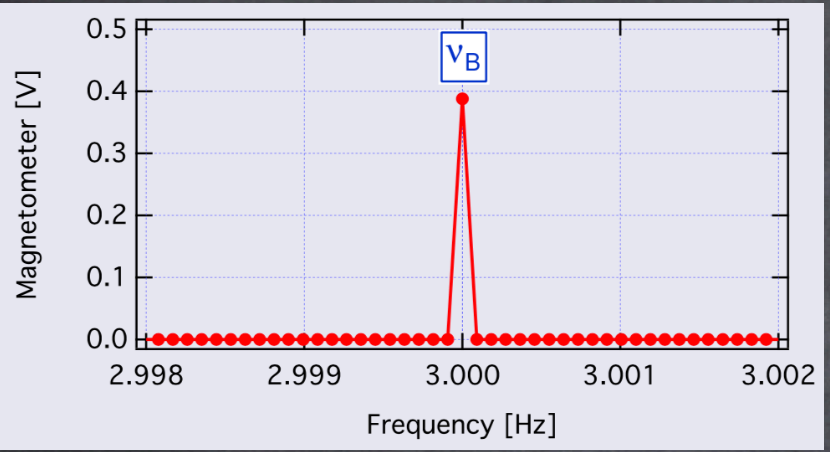


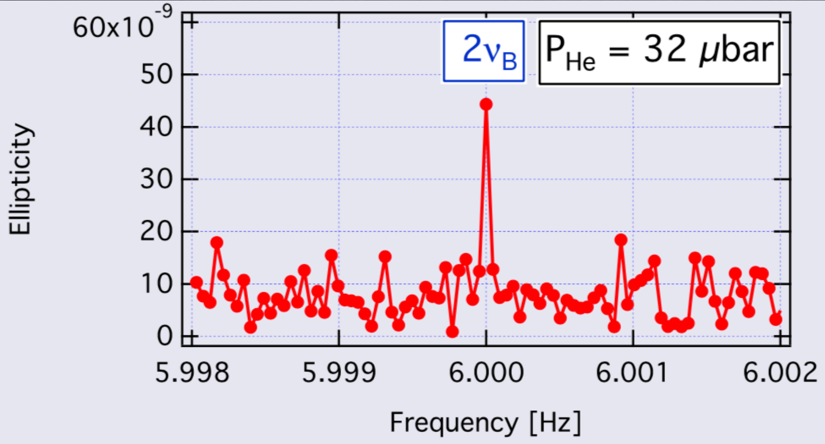
PVLAS output end



Calibration with He - Cotton Mouton effect

Magnetic field and ellipticity signals for
T = 5.8 hours and P = 32μbar of He

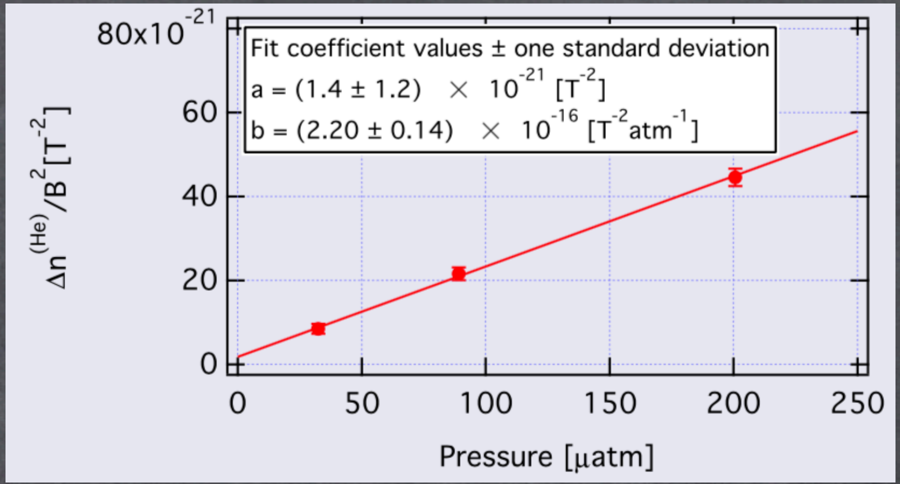


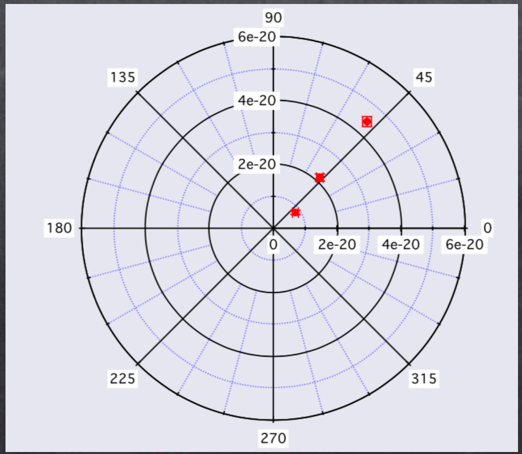


The signal occupies a single bin in the ellipticity FFT spectrum

$$\Psi_{\mathrm{gas}} = N\pi\frac{L}{\lambda}\Delta n_{\mathrm{c.m.}}B^2 p\sin 2\vartheta$$

Amplitude and phase calibration

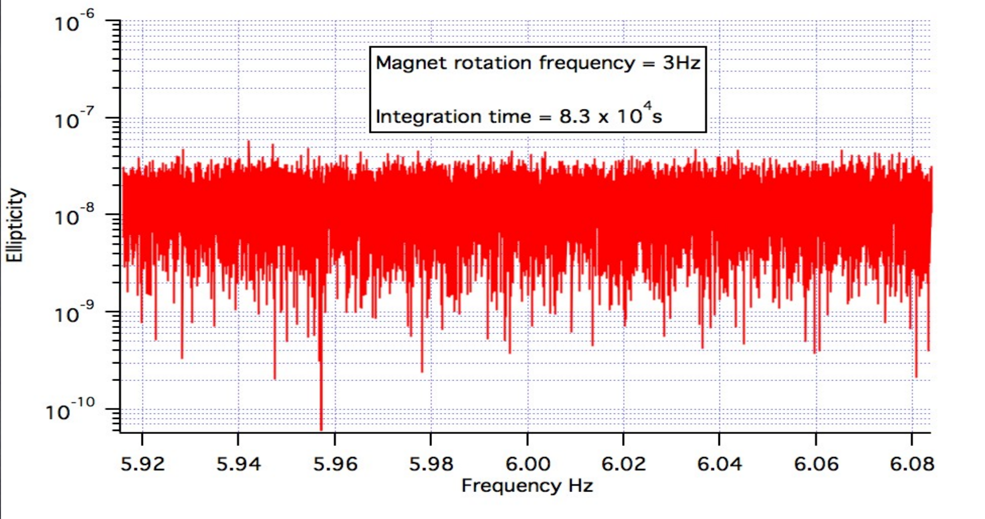




Measurements as a function of low pressure of Helium:
P = (32, 97, 198) μbar

Example of vacuum birefringence result

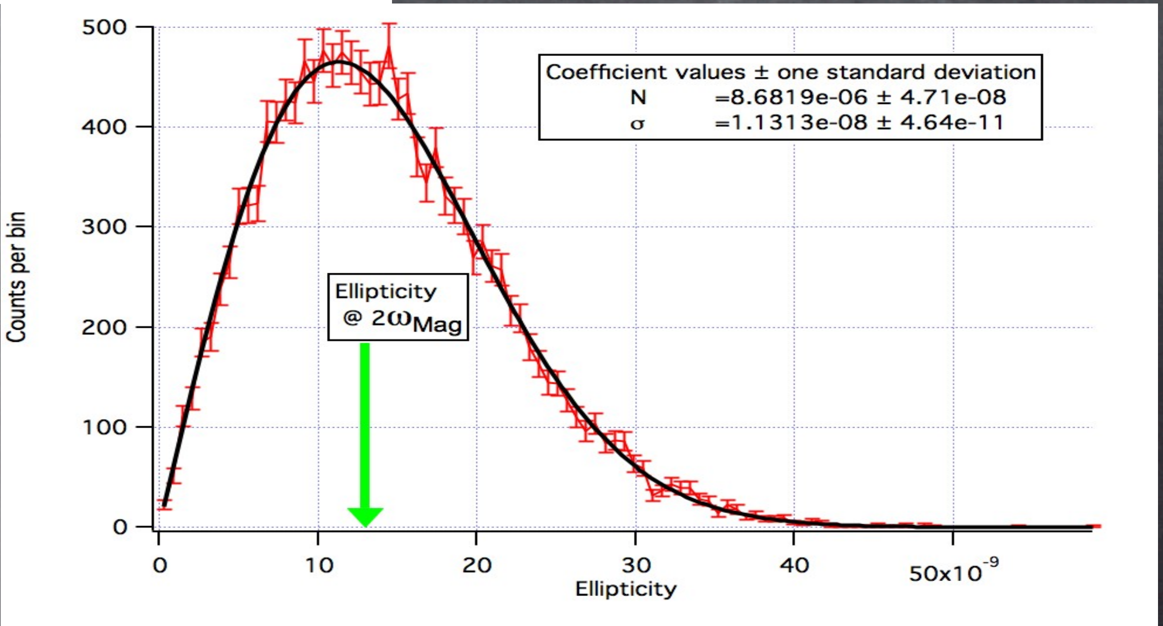
FFT Spectrum of data around signal frequency



Distribution of noise is a Rayleigh function

$$P(r) = N\frac{r}{\sigma_\psi^2}e^{-\frac{r^2}{2\sigma_\psi^2}}$$

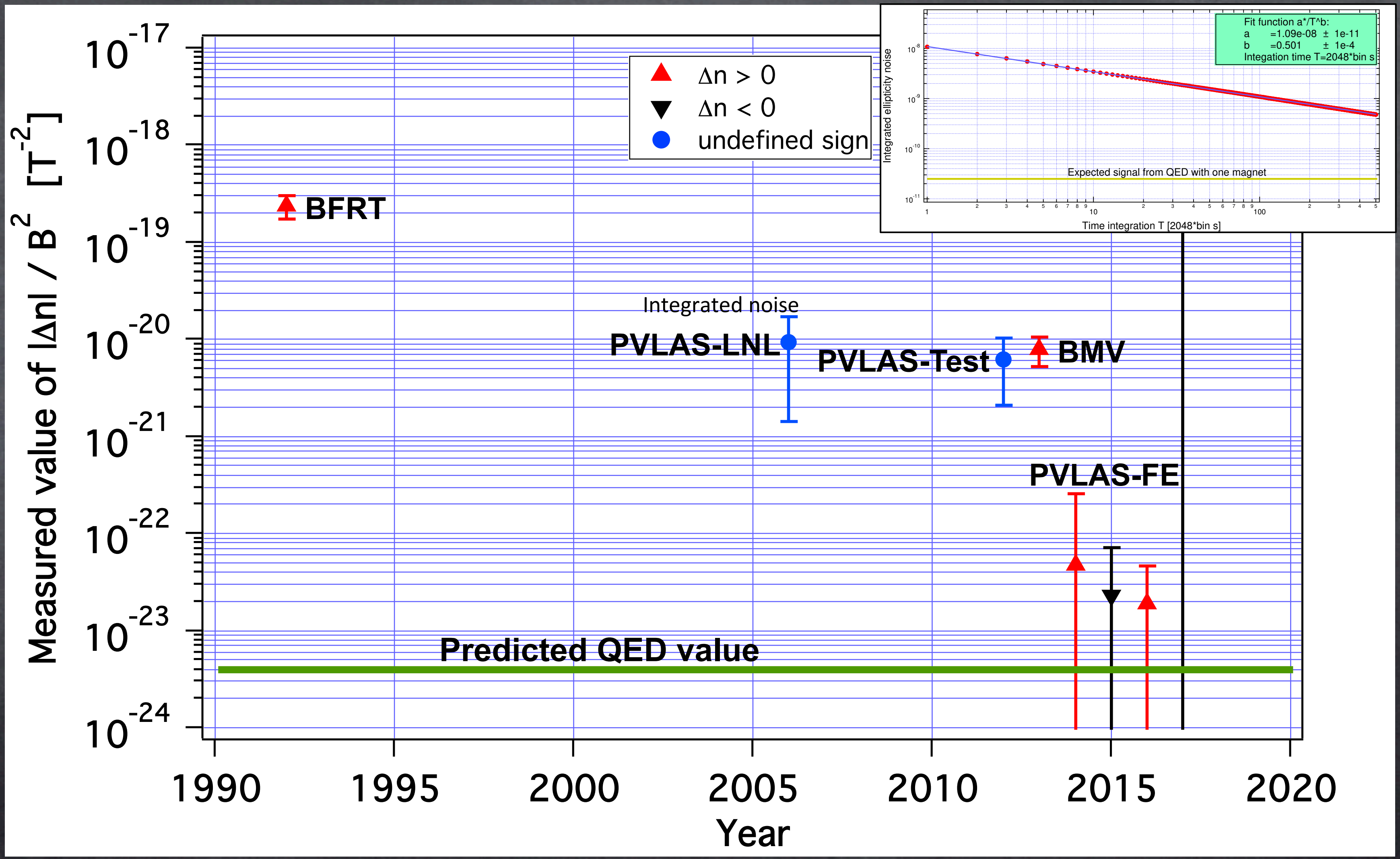
$\sigma_\psi = 1.1\ 10^{-8}$



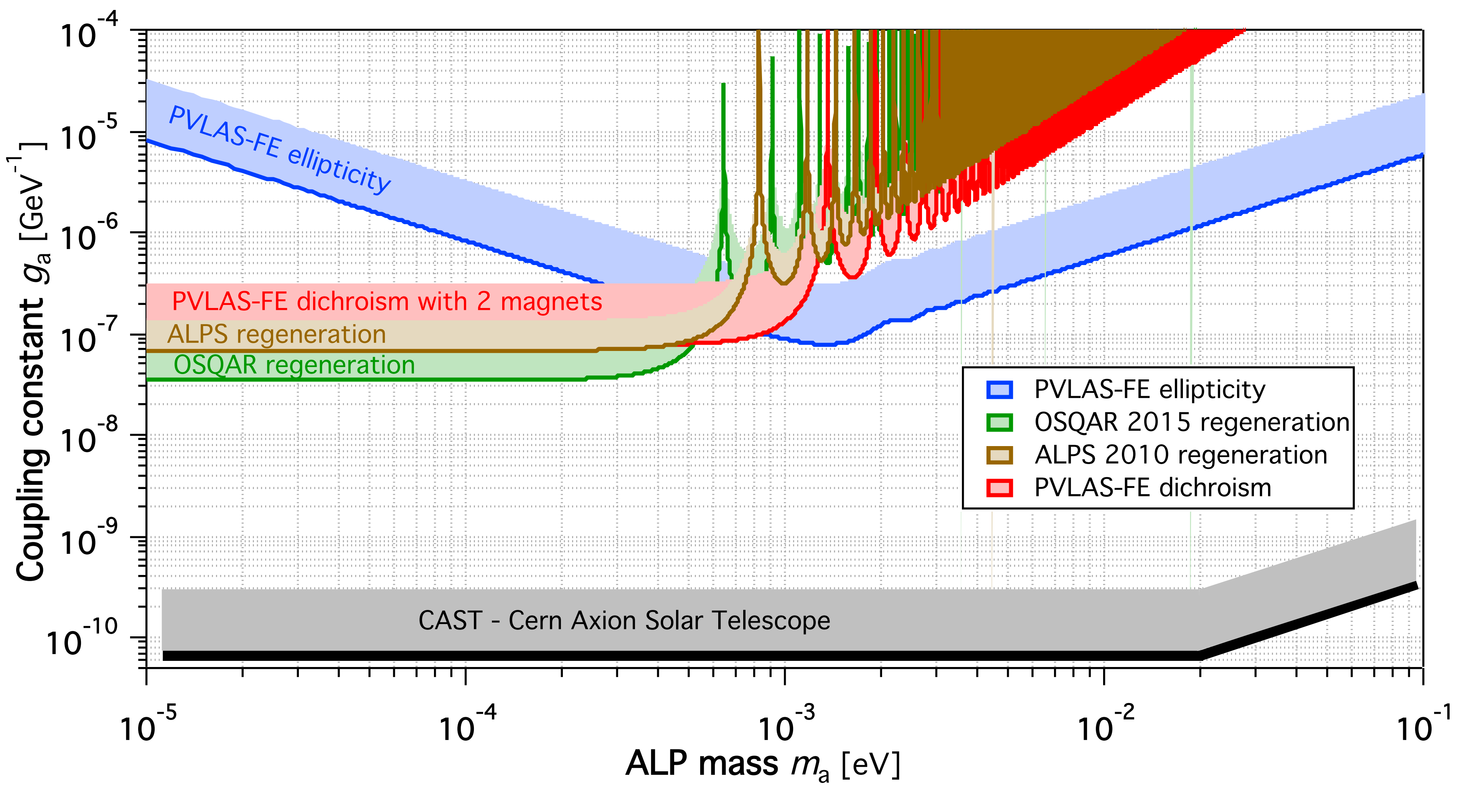
- One determines the phase and amplitude at $2\nu_B$
- Project result along physical axis

Vacuum magnetic birefringence

- 2016 PVLAS point corresponds to an integration $T \approx 5 \cdot 10^6$ s.
- Could not overcome the gap by integrating longer
- The use of permanent magnets allowed detailed debugging.



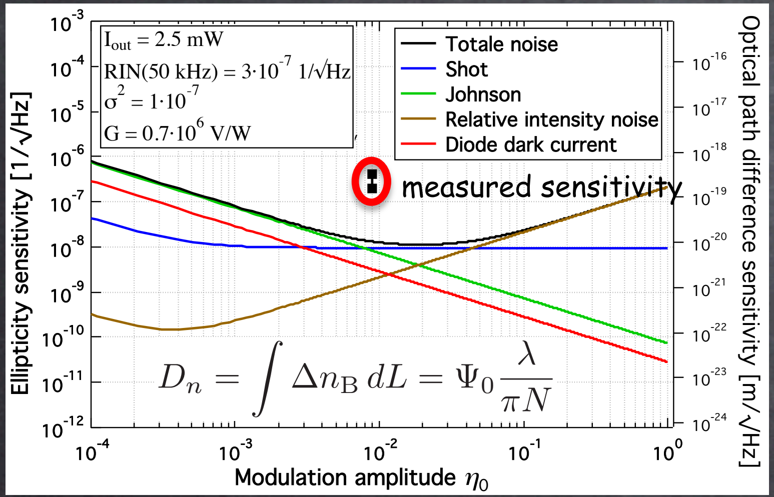
ALP laboratory limits

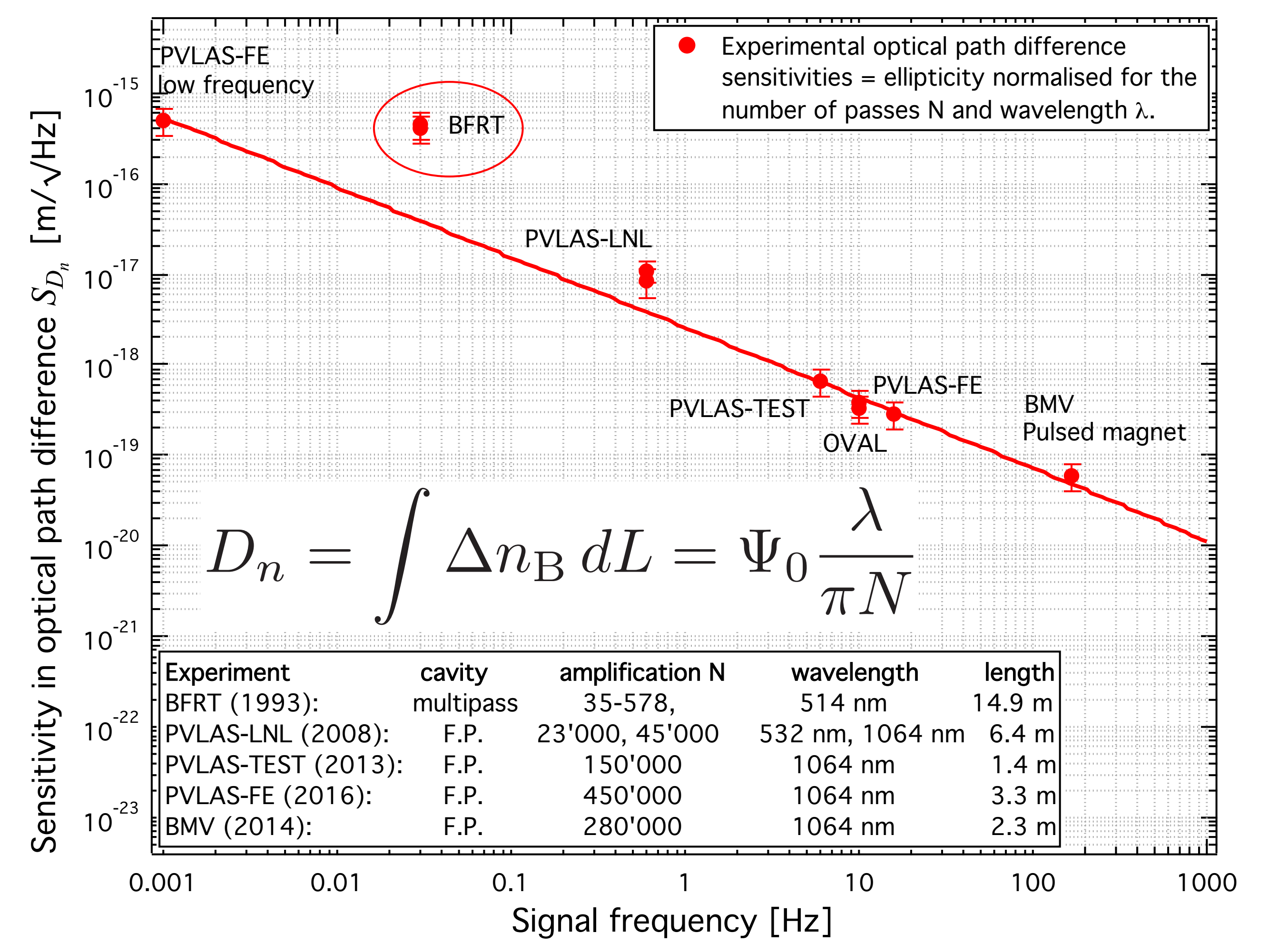


Excluded regions are above the curves

Intrinsic noise?

Sensitivity in optical path difference D_n between two perpendicular polarizations





Experiment	cavity	amplification N	wavelength	length
BFRT (1993):	multipass	35-578,	514 nm	14.9 m
PVLAS-LNL (2008):	F.P.	23'000, 45'000	532 nm, 1064 nm	6.4 m
PVLAS-TEST (2013):	F.P.	150'000	1064 nm	1.4 m
PVLAS-FE (2016):	F.P.	450'000	1064 nm	3.3 m
BMV (2014):	F.P.	280'000	1064 nm	2.3 m

Updated graph from G. Zavattini et al. Eur. Phys. J. C (2016) 76:294

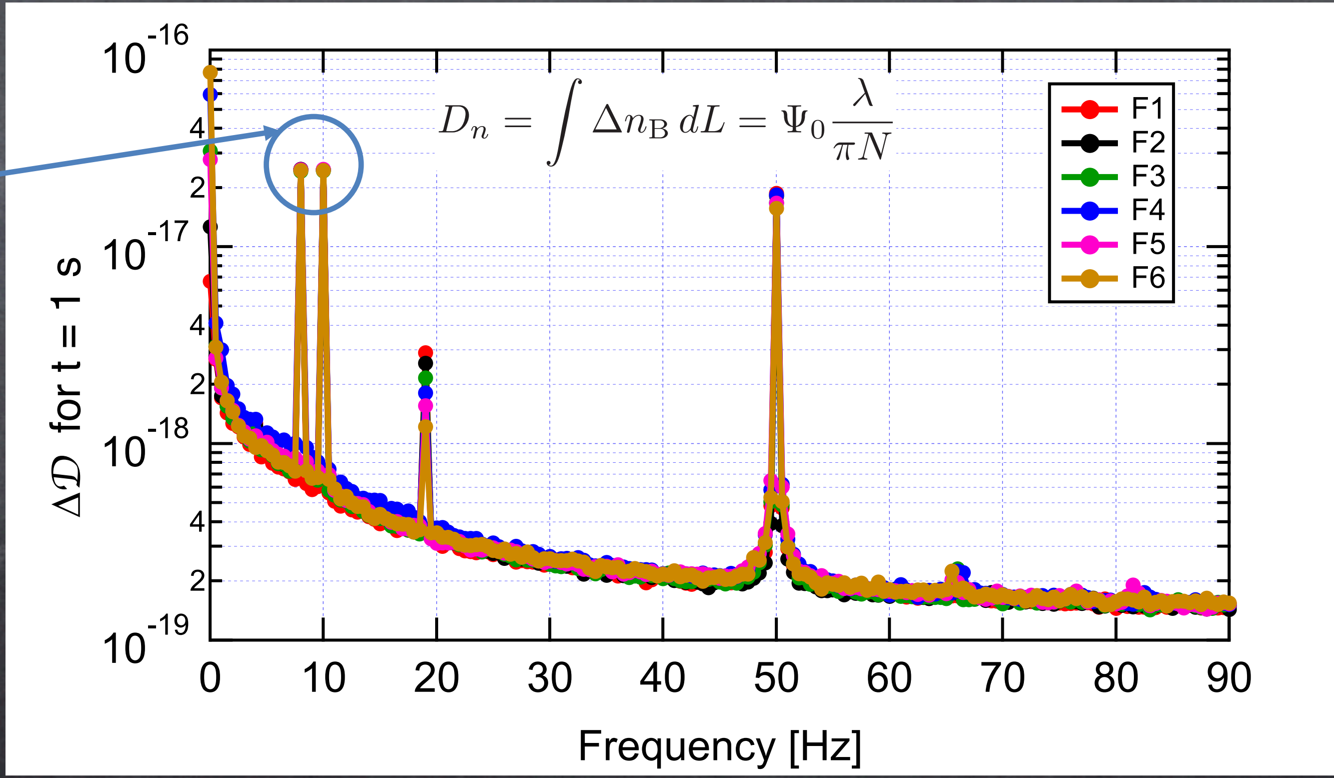
BFRT: R. Cameron et al. PRD, **47** (1993) 3707
PVLAS-LNL: M. Bregant et al. PRD, **78** (2008) 032006
PVLAS-TEST: F. Della Valle et al. NJP, **15** (2013) 053026
PVLAS-FE: F. Della Valle et al. EPJC, **76:24** (2016) 1
BMV: A. Cadène et al. EPJD, **68:16** (2014) 1

Sensitivity in D_n does not depend on finesse

Intrinsic noise

- We determined the optical path difference noise and Cotton-Mouton signals as a function of the finesse.
- Introduced controlled extra loses $p \approx 10^{-5}$ in the cavity by clipping the beam.
- Finesse ranged: 250’000 – 690’000

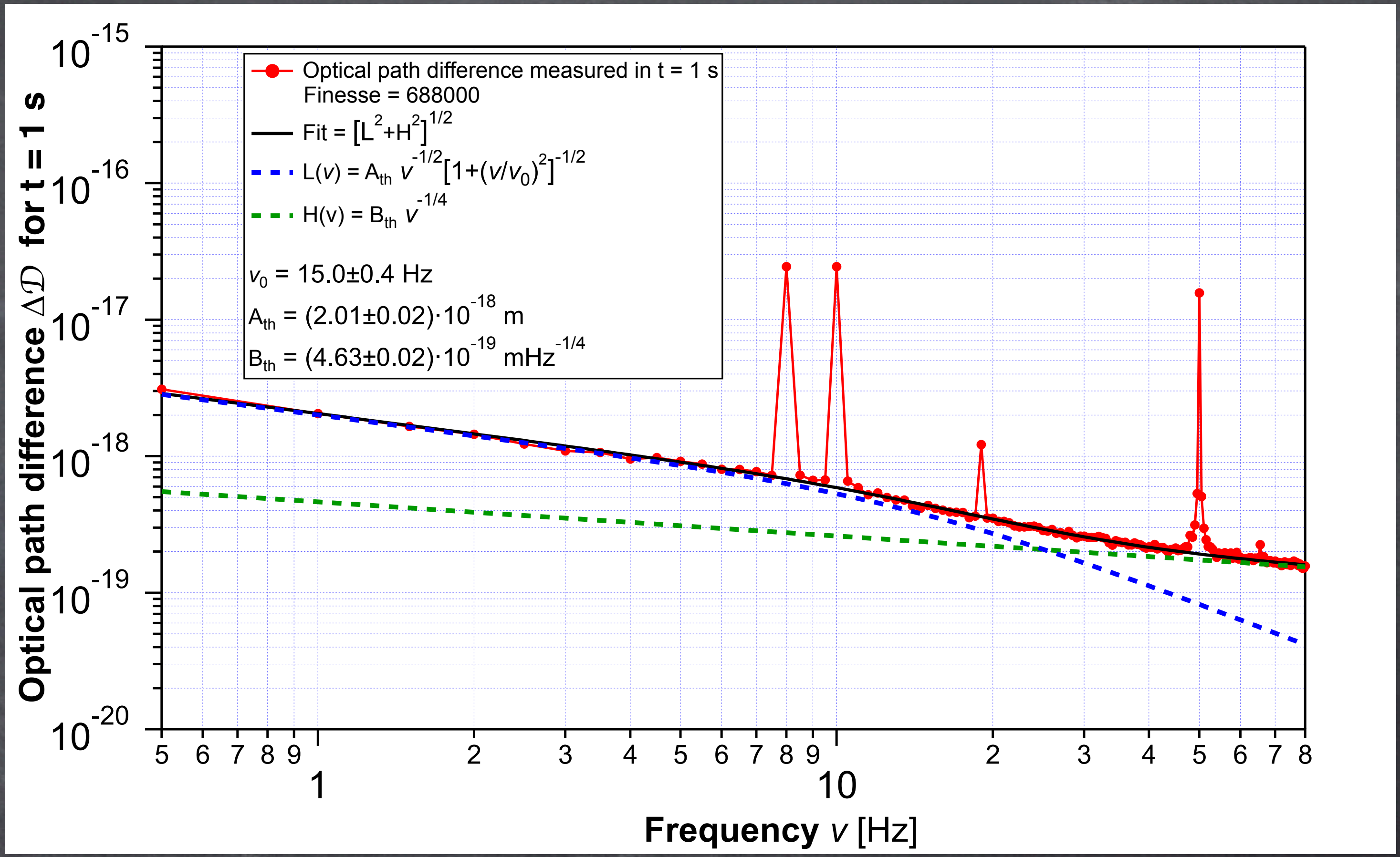
Cotton-Mouton signals



Noise and Cotton-Mouton optical path difference signals are independent of the finesse

Noise origin

A. Ejlli et al. Physics Reports 871 (2020) 1–74



We believe the noise is of thermal origin. Local temperature fluctuations generate local stress fluctuations. Through the stress optic coefficient this generates birefringence fluctuations: thermo-elastic noise. Estimates indicate that the Ta_5O_2 layers dominate

$B_{\rm th}$ describes thermo-elastic effect in the coating

$A_{\rm th}$ describes Brownian noise?

$$f(\nu) = \sqrt{\left(\frac{A_{\rm th}\nu^{-1/2}}{\sqrt{1+(\nu/\nu_0)^2}}\right)^2 + \left(B_{\rm th}\nu^{-1/4}\right)^2}$$

How to beat the noise

- Increase the frequency of the signal by rotating faster
  - $S_{\mathcal{D}_n} \propto \nu^{\alpha}$ with $\alpha \approx -0.7$
  - Maybe could improve by a factor 2 with the PVLAS apparatus
- Increase the signal: B^2L of magnet
  - Only real option is to use superconducting magnets
  - One LHC magnet has B^2L = 1200 T^2m. At present we have 10 T^2m.
  - Superconductor magnets cannot be modulated at ≈ 10 Hz
- Change origin of modulation
  - Rotate the polarization inside the field
  - … But must be kept fixed on the mirrors.

Future

VMB@CERN LoI

R. Ballou[1)], F. Della Valle[2)], A. Ejlli[3)], U. Gastaldi[4)], H. Grote[3)], Š. Kunc[5)], K. Meissner[6)], E. Milotti[7)], W.-T. Ni[8)], S.-s. Pan[9)], R. Pengo[10)], P. Pugnat[11)], G. Ruoso[10)], A. Siemko[12)], M. Šulc[5)] and G. Zavattini[13)]*

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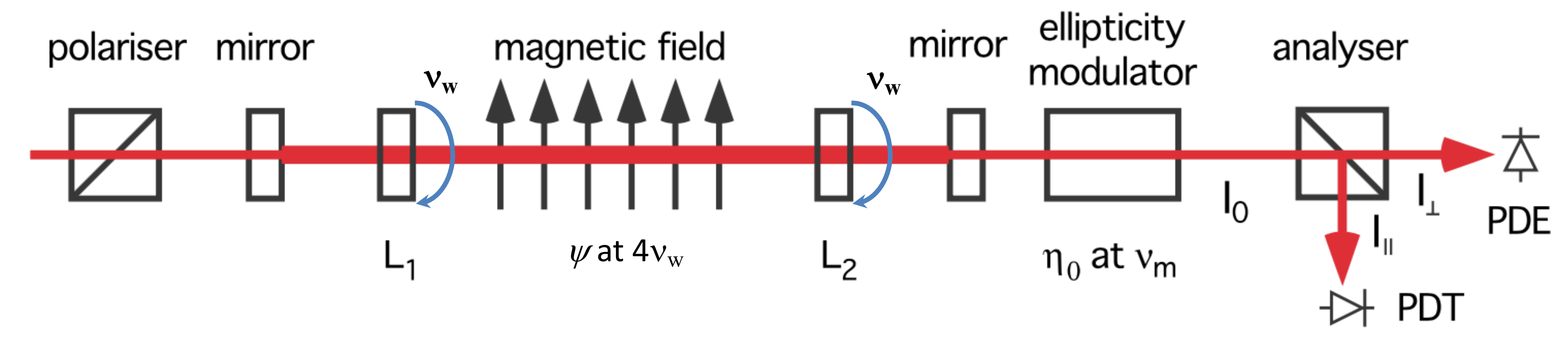
R. Ballou, et al., Letter of Intent to measure Vacuum Magnetic Birefringence: the VMB@CERN experiment, Tech. Rep. CERN-SPSC-2018-036. SPSC-I-249, CERN, Geneva (2018). https://cds.cern.ch/record/2649744

Separate magnet from modulation

Polarization modulation scheme

- Rotate polarization inside the magnet
- Fix polarization on mirrors to avoid mirror birefringence signal
- Insert two co-rotating half wave plates @ ν_W with a fixed relative angle $\Delta\phi$
  - Total losses ≤ 0.4% (commercial). Maybe 10 times lower is possible
  - Maximum finesse ≈ 1000 (with ≤ 0.4% losses)



G. Zavattini et al. Eur. Phys. J. C (2016) 76:294

Signal and possible problems

G. Zavattini et al. Eur. Phys. J. C (2016) 76:294

$$I(t) = I_{\rm out}\left\{\eta(t)^2 + 2\eta(t)N\left[\psi_0 \sin 4\phi(t) + \alpha_1 \sin 2\phi(t) + \alpha_2 \sin(2\phi(t) + 2\Delta\phi)\right]\right\}$$

Signal appears a the 4th harmonic of $\nu_{\rm w}$. $\alpha_{1,2}$ are the wave phase errors from π.

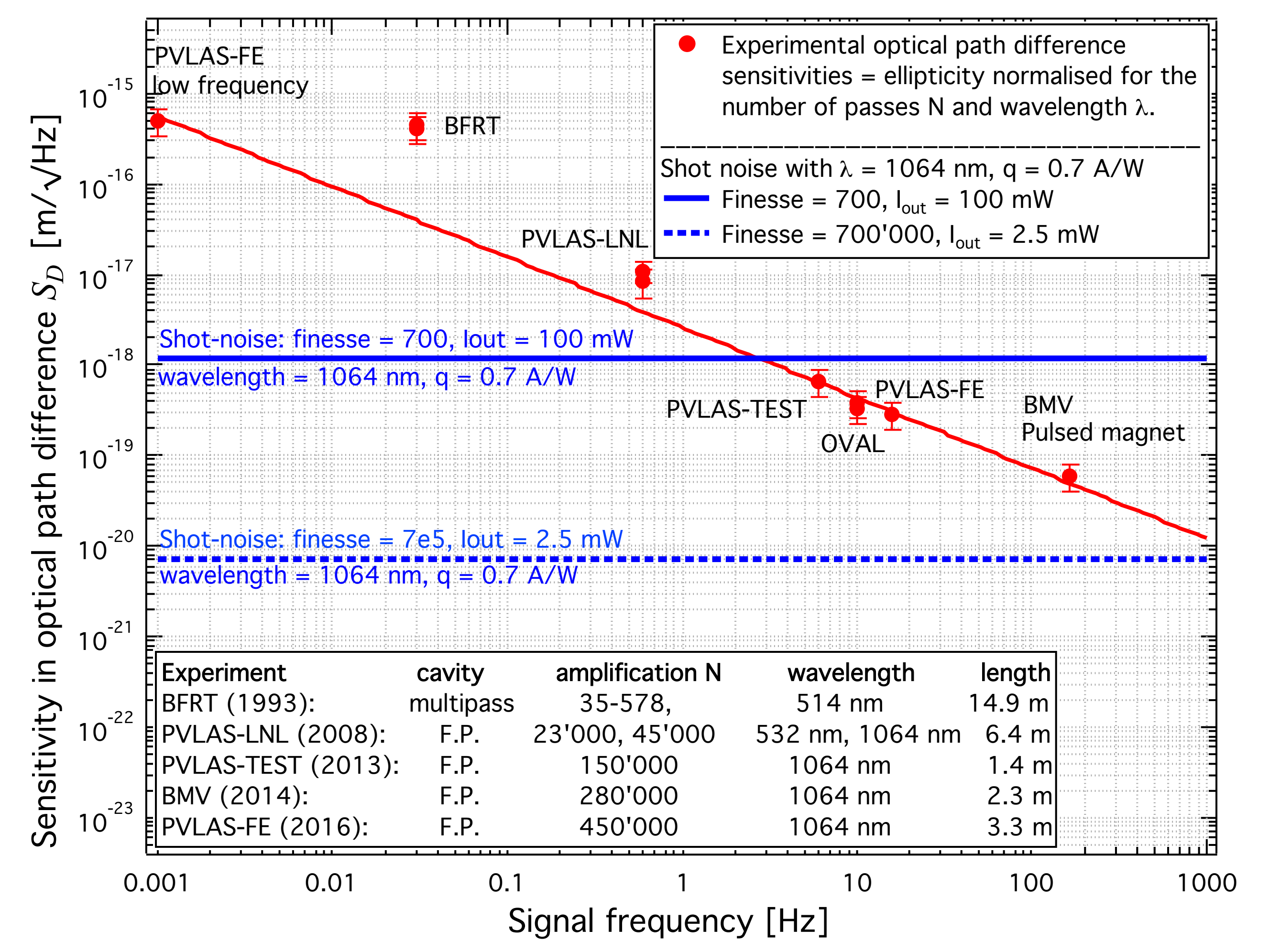
Main issues:

- Control the transverse position of each rotating HWP independently to reduce wobble and intrinsic wave plate defects:
  - use a frequency doubled laser for which a HPW becomes a ≈ FWP.
- Control the relative angular phase between the two rotating HWP:
  - use a Faraday rotator to measure it and control it with a feedback.
- On going work in Ferrara where the optical test installation is in progress.

What sensitivity could be reached?

Sensitivity in optical path difference D_n between two perpendicular polarizations

$D_n \approx 10^{-18}$ m/√Hz goal sensitivity



Updated graph from G. Zavattini et al. Eur. Phys. J. C (2016) 76:294

VMB@CERN with 1 LHC magnet

- Signal $$D_n = 3A_e B^2 L = 4.8 \times 10^{-21}\ \mathrm{m}$$

- Intrinsic noise $$S_{D_n}^{(\mathrm{intrinsic})} \approx 2.6 \times 10^{-18} \nu^{-0.77}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$$

- Shot-noise $$S_{D_n}^{(\mathrm{shot})} = \sqrt{\frac{e}{I_0 q}} \frac{\lambda}{\pi N}$$

- Maximum measurement time $$T = \left(\frac{S_{D_n}}{D_n}\right)^2 \lesssim 10^6\ \mathrm{s}$$

- LHC example: $B^2 L = 1200\ \mathrm{T^2 m}$; $S_{D_n} = 10^{-18}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$ @ 3 Hz

$$\Rightarrow T = 12\ \mathrm{h}$$

Conclusions

- The PVLAS - FE experiment has set the best limits on vacuum magnetic birefringence.
- The goal of measuring VMB is still out of reach
- Rotating half wave-plates *inside* a Fabry-Perot cavity together with a spare LHC dipole could be a viable solution to separating the external magnetic field intensity from the modulation frequency
- Technique must be tested
  - Extinction ratio?
  - Extra wide band noise?
  - Maximum finesse? F = 850 achieved; F ≈ 10'000?
- Intrinsic HWP defects and rotation errors may be a limit to this idea. Test are underway in Ferrara
- With a sensitivity of $D_n \approx 10^{-18}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$ and 1 LHC magnet, vacuum magnetic birefringence could be measured with S/N = 1 in about 1 day.
- LoI has been submitted to CERN: CERN-SPSC-2018-036/SPSC-I-249 It is a joint effort between past vacuum magnetic birefringence experiments + some LIGO people.

Thank you!