What Lattice QCD can do for experiment

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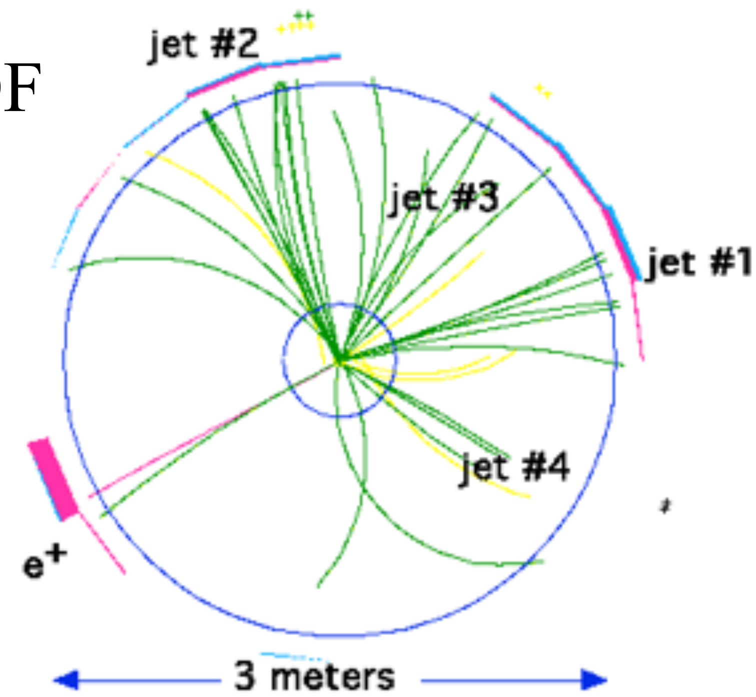
HPQCD collaboration

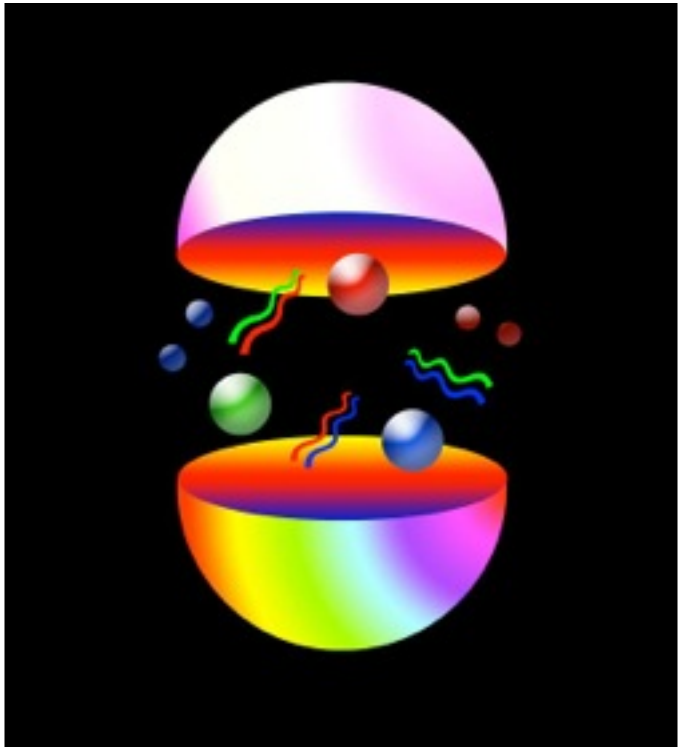
Birmingham Nov 2014

QCD is a key part of the Standard Model but quark confinement is a complication/interesting feature.

CDF



Cross-sections calculated at high energy using QCD pert. th. with ~3% errors. Also parton distribution function and hadronisation uncertainties.

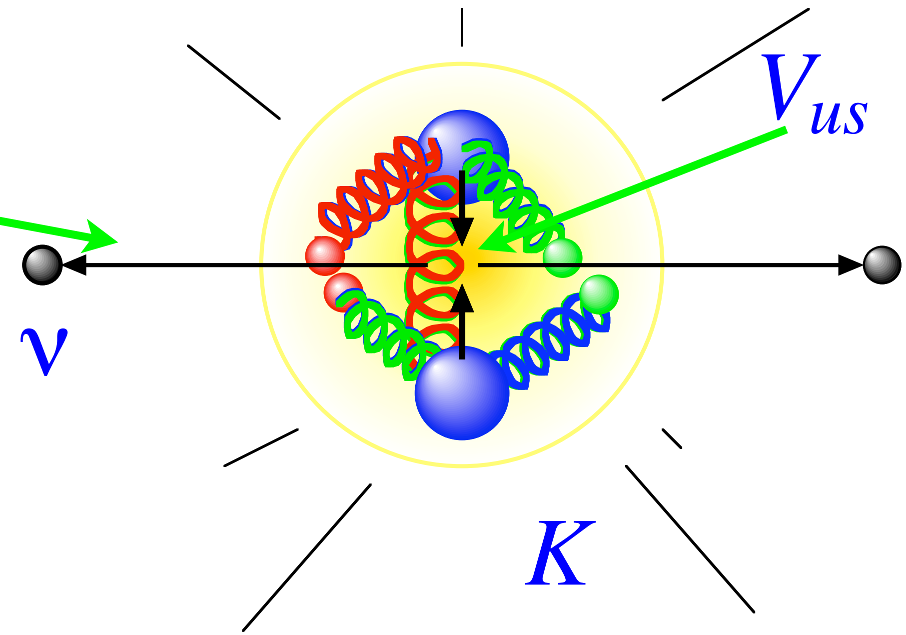


But (some) properties of hadrons much more accurately known and calculable in lattice QCD - can test SM and determine parameters very accurately (1%).

Weak decays probe meson structure and quark couplings

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to l\nu & K \to l\nu & B \to \pi l\nu \\ & K \to \pi l\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \to l\nu & D_s \to l\nu & B \to D l\nu \\ D \to \pi l\nu & D \to K l\nu & \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d | \overline{B}_d \rangle & \langle B_s | \overline{B}_s \rangle & \end{pmatrix}$$

CKM matrix

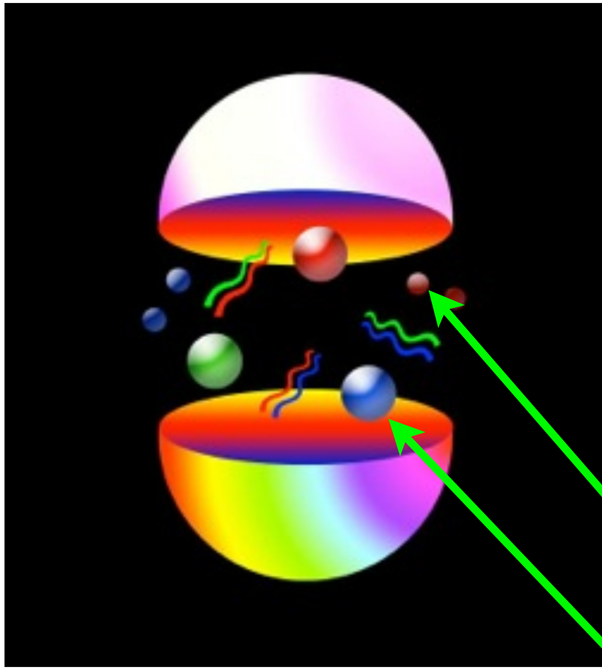


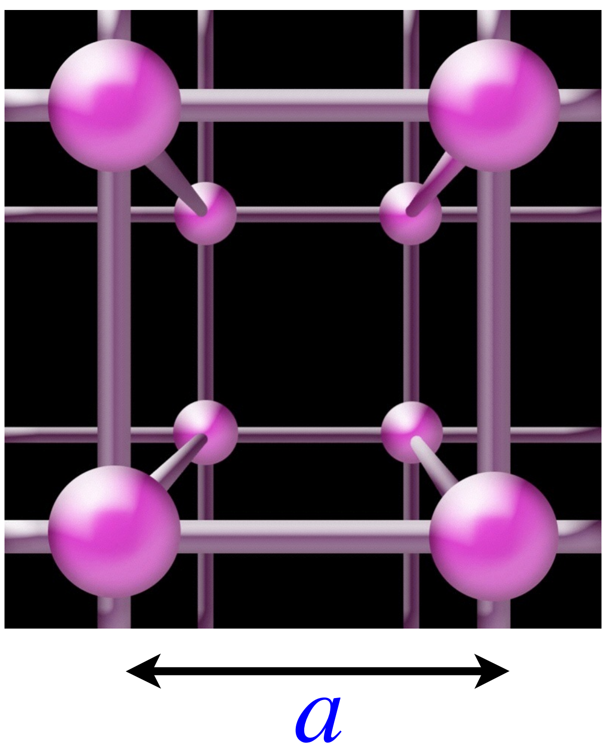
$$Br(M \to \mu\nu) \propto V_{ab}^2 f_M^2$$

Expt = CKM x theory(QCD)

Need precision lattice QCD to get accurate CKM elements to test Standard Model (e.g. is CKM unitary?).

If V_{ab} known, compare lattice to expt to test QCD





Lattice QCD = fully nonperturbative, based on Path Integral formalism

basic integral $\int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\overline{\psi}\, \exp(-\int \mathcal{L}_{QCD} d^4x)$

- Generate sets of gluon fields for Monte Carlo integrn of Path Integral (inc effect of u, d, s (+ c) sea quarks)
- Calculate averaged “hadron correlators” from valence q props.
- Fit as a function of time to obtain masses and simple matrix elements
- Determine a and fix m_q to get results in physical units.
- extrapolate to $a = 0, m_{u,d} = phys$ for real world **now at m_{phys}**

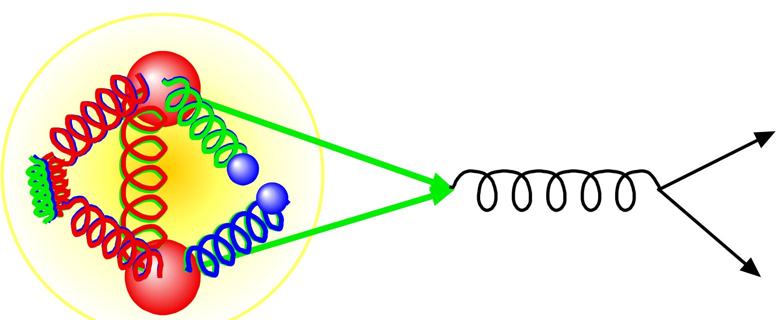
Hadron correlation functions ('2point functions') give masses and decay constants.

$$\langle 0|H^\dagger(T)H(0)|0\rangle = \sum_n A_n e^{-m_n T} \overset{T}{\underset{\text{large}}{\rightarrow}} A_0 e^{-m_0 T}$$

masses of all hadrons with quantum numbers of H

$$A_n = \frac{|\langle 0|H|n\rangle|^2}{2m_n} = \frac{f_n^2 m_n}{2}$$

decay constant parameterises amplitude to annihilate - a property of the meson calculable in QCD. Relate to experimental decay rate.



1% accurate experimental info. for f and m for many mesons! Need accurate determination from lattice QCD to match

Darwin@Cambridge, part of STFC's HPC facility for theoretical particle physics and astronomy - DiRAC II

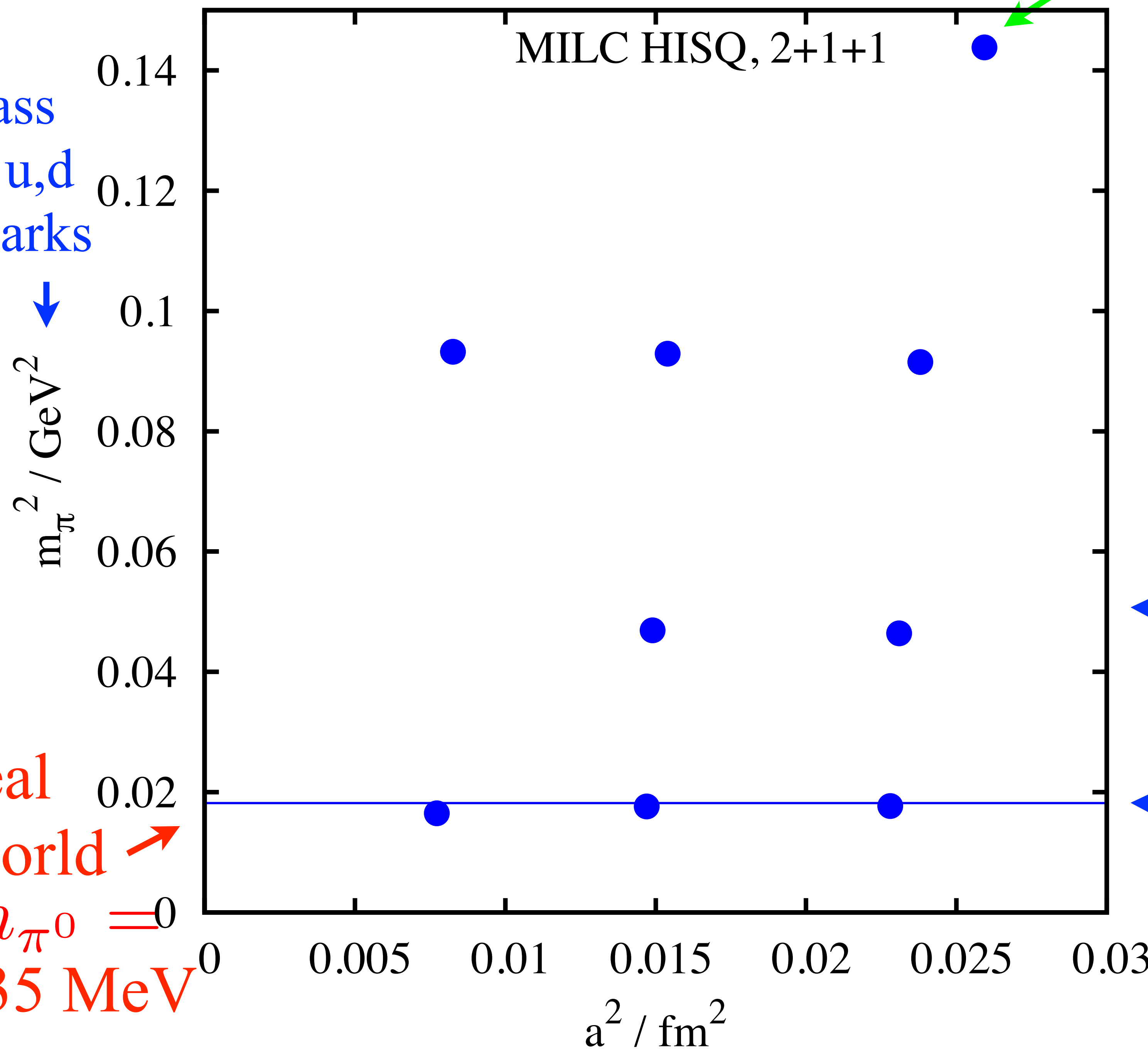
State-of-the-art commodity cluster: 9600 Intel Sandybridge cores, infiniband interconnect, fast switch and 2 Pbytes storage

DiRAC

www.dirac.ac.uk

Allows us to calculate quark propagators rapidly and store them for flexible re-use.

Example parameters for calculations now being done with 'staggered' quarks.



mass of u,d quarks

real world

$m_{\pi^0} =$ 135 MeV

"2nd generation" lattices inc. c quarks in sea

HISQ = Highly improved staggered quarks - very accurate discretisation

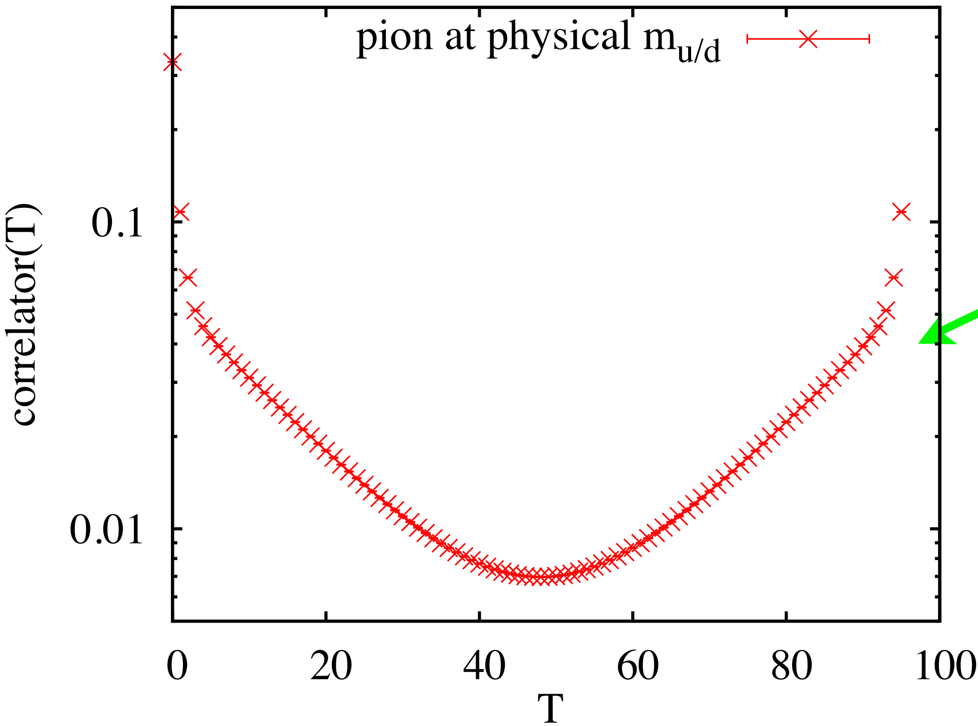
E.Follana et al, HPQCD, hep-lat/ 0610092.

$m_{u,d} \approx m_s/10$

$m_{u,d} \approx m_s/27$

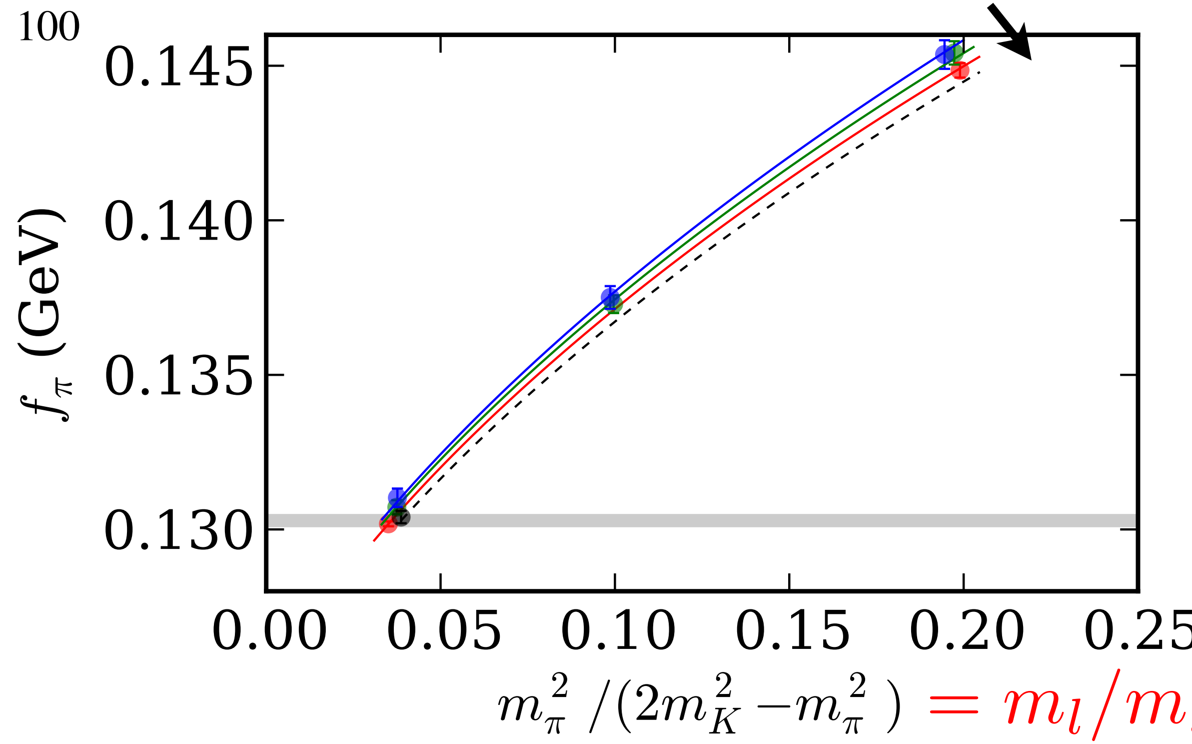
Volume: $m_\pi L > 3$

Example (state-of-the-art) calculation



Extract meson mass and amplitude=decay constant from correlator for multiple lattice spacings and $m_{u/d}$.
Very high statistics

Extrapolation in a

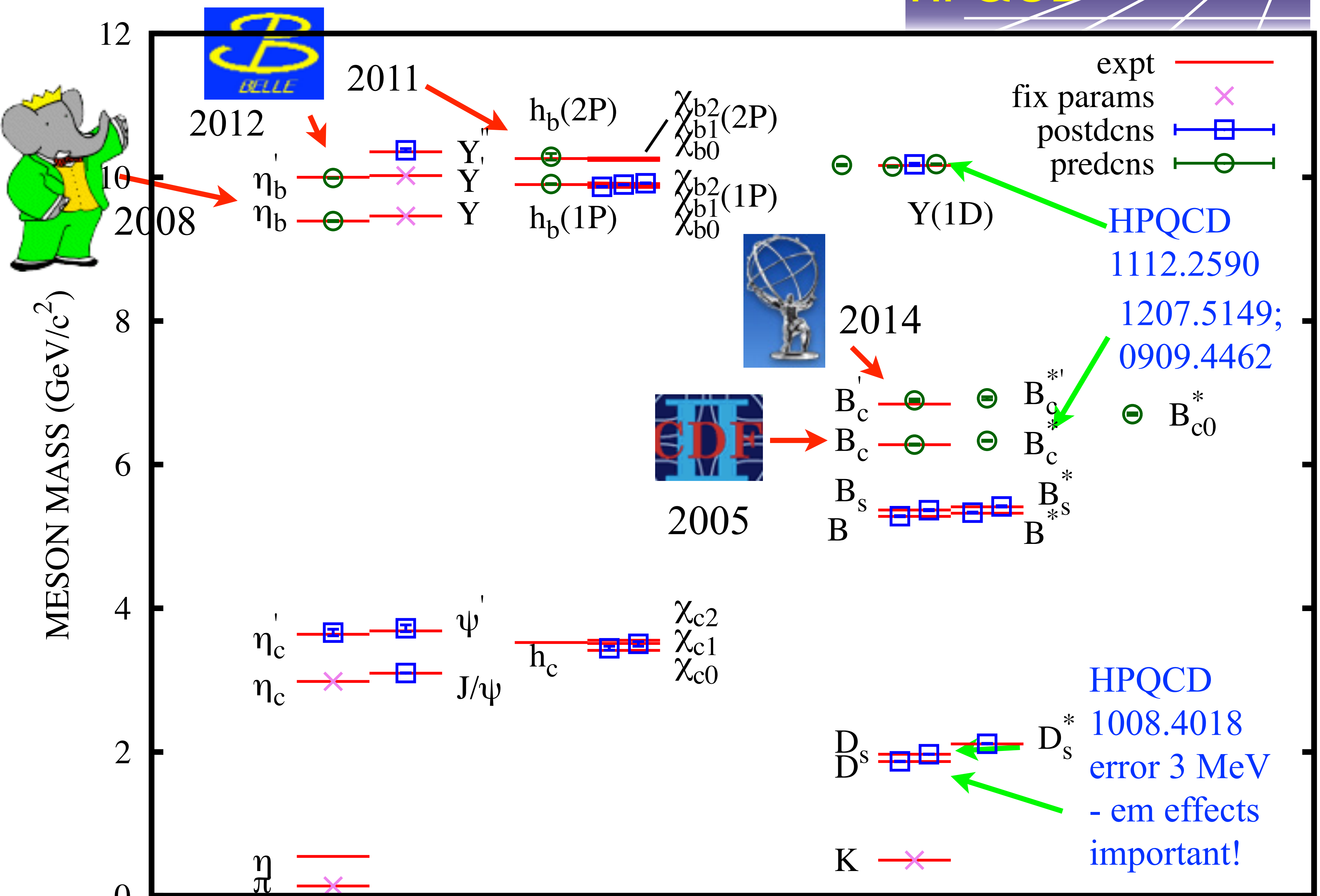


Convert decay constant to GeV units using w_0 to fix relative lattice spacing. Very small discretisation errors.

R. Dowdall et al, HPQCD, 1303.1670.

The gold-plated meson spectrum

HPQCD



older predcns: I. Allison et al, hep-lat/0411027, A. Gray et al, hep-lat/0507013

Lattice QCD is best method to determine quark masses

$m_{q,latt}$ determined very accurately by fixing a meson mass to be correct. e.g. for m_c fix M_{η_c}

Issue is conversion to the $\overline{MS}$ scheme

masses important for Higgs cross-sections

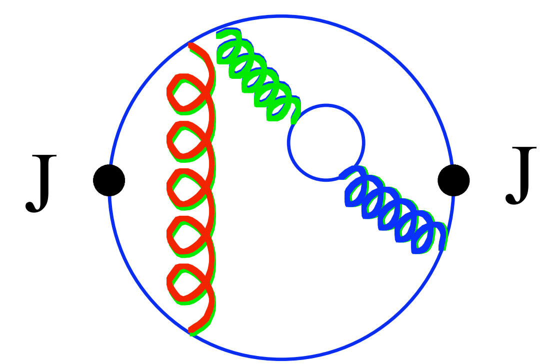
- Direct method

$$m_{\overline{MS}}(\mu) = Z(\mu a) m_{latt}$$

Calculate Z perturbatively or partly nonperturbatively.

- Indirect methods: (after tuning m_{latt}) match a quantity from lattice QCD to contnm pert. th. in terms of $\overline{MS}$ mass

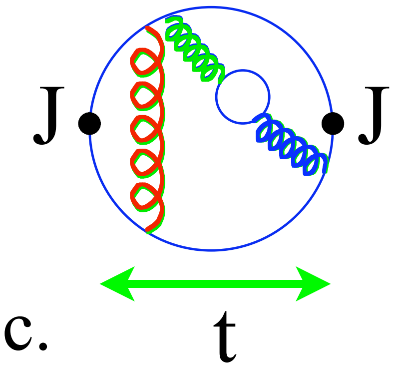
e.g. q^2-derivative moments of current-current correlators (vac. pol.function) for heavy quarks known through α_s^3 .
Calc. on lattice as time-moments of 'local' meson correlation function



Chetyrkin et al, 0907.2110
HPQCD + Chetyrkin et al, 0805.2999, C. Mcneile et al, HPQCD,1004.4285

Most accurate to use pseudoscalar correlator time-moments:

$$G(t) = a^6 \sum_{\vec{x}} (am_c)^2 < 0|j_5(\vec{x},t) j_5(0,0)|0 >$$



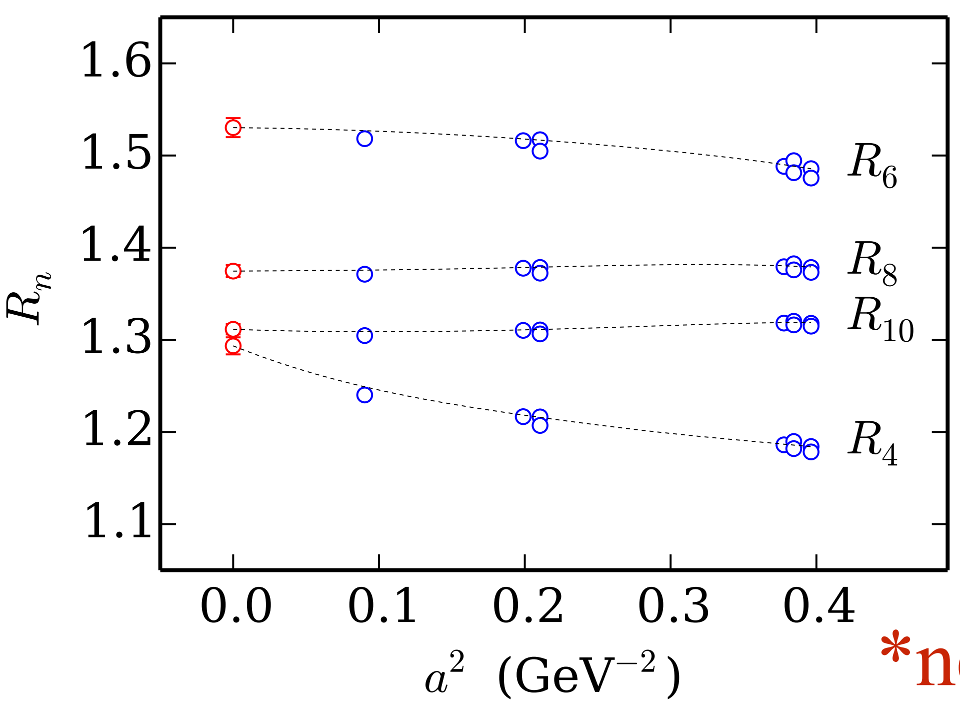
$$G_n = \sum_t (t/a)^n G(t)$$

ratio to results with no gluon field improves disc. errors

$$R_{n,latt} = G_4/G_4^{(0)} \quad n = 4$$

$$= \frac{am_{\eta_c}}{2am_c}(G_n/G_n^{(0)})^{1/(n-4)} \quad n = 6, 8, 10 \ldots$$

extrapolate first 4 moments to a=0 and fit to contnm pert. th. gives $\overline{m}(\mu)$ AND $\alpha_s(\mu)$



From 2+1 configs:

$$m_b(m_b) = 4.164(23)\text{GeV}$$

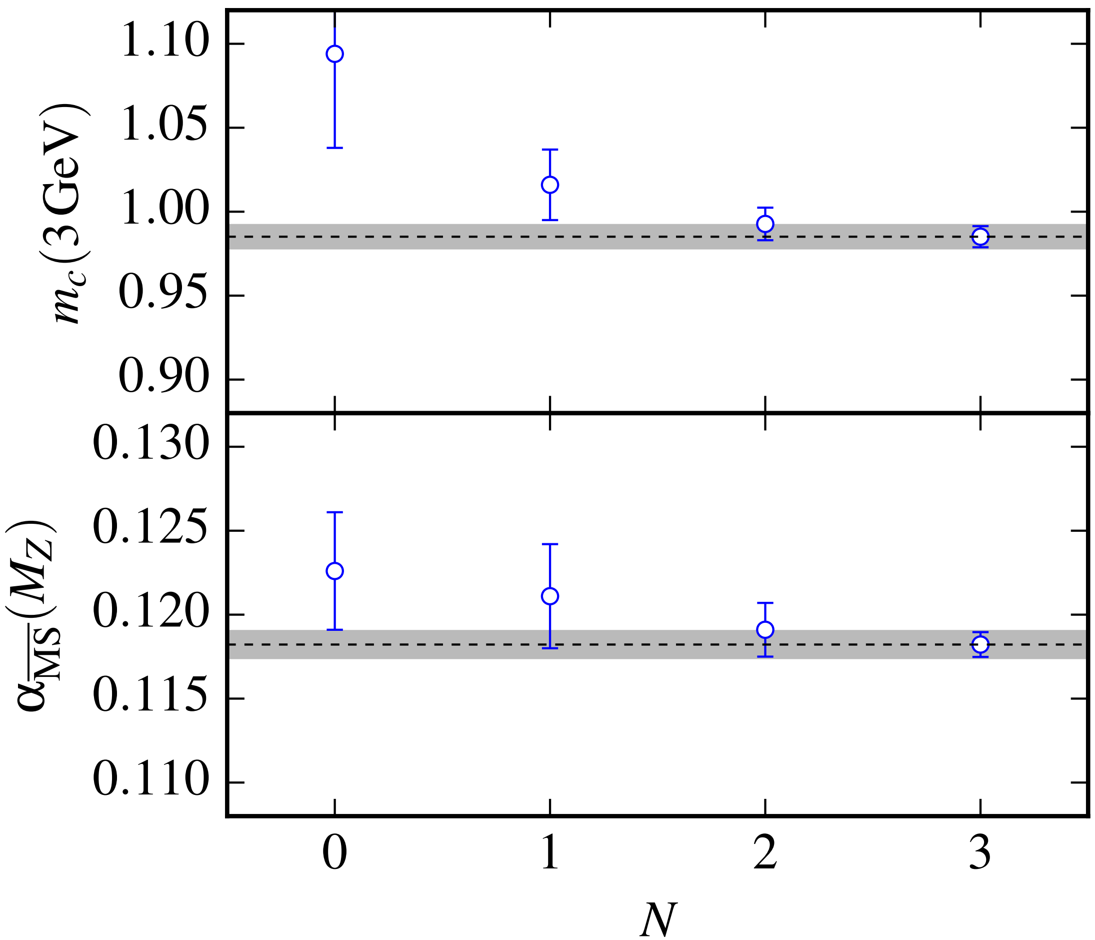
$$m_c(m_c) = 1.273(6)\text{GeV}$$

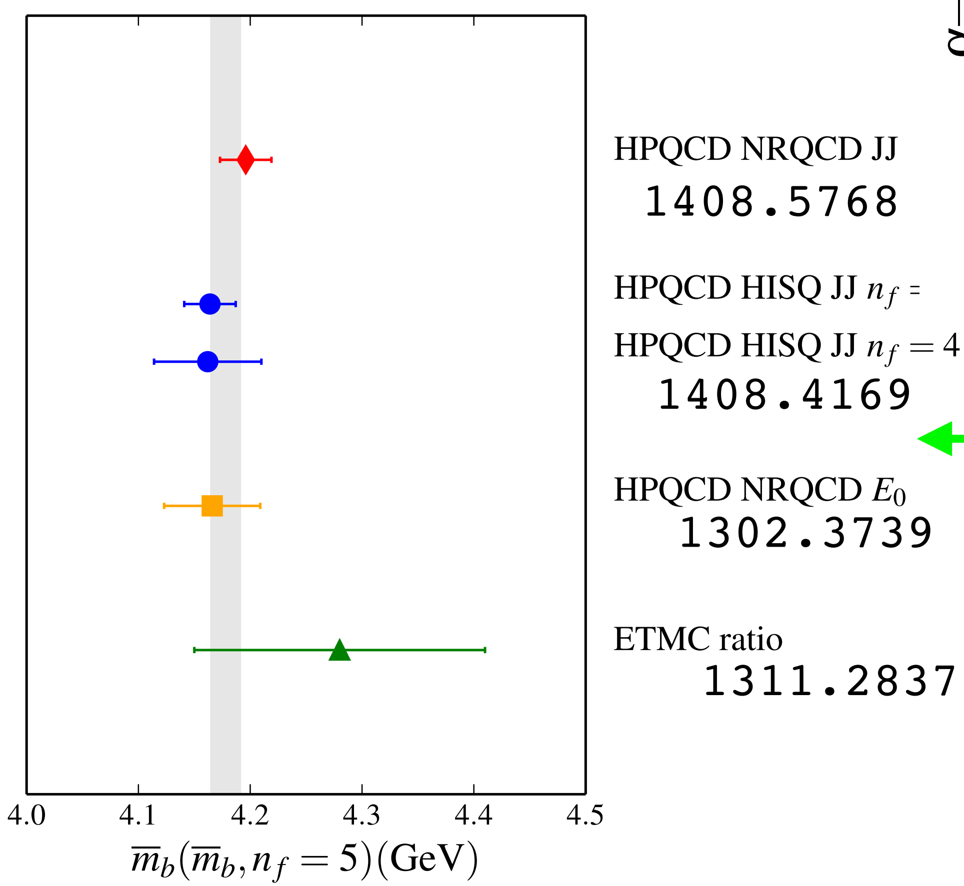
$$\alpha_s(M_z) = 0.1183(7)$$

new 2+1+1 results agree:1408.4169

Improvement in result clear as more orders added in contnm pert. theory.

HPQCD,1408.4169





Different lattice methods for m_b agree. Weighted average (grey band): 4.178(14) GeV

1404:0319: impact on accuracy of $H \to b\overline{b}$

Quark mass ratios

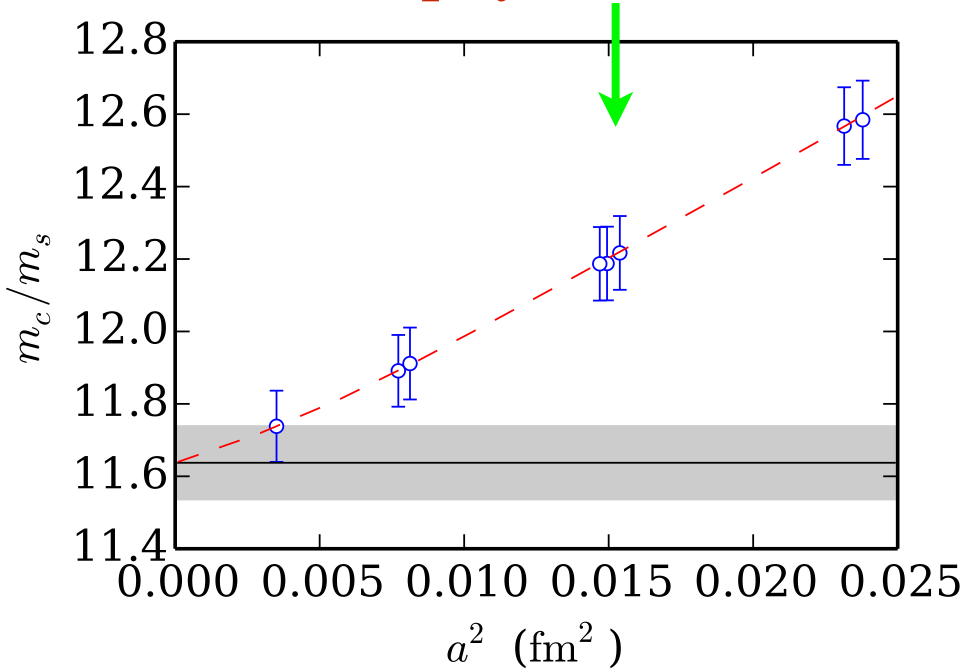
HPQCD: 0910.3102; 1004.4285,1408.4169

Obtained directly from lattice QCD if same quark formalism is used for both quarks.
Ratio is at same μ and for same nf.

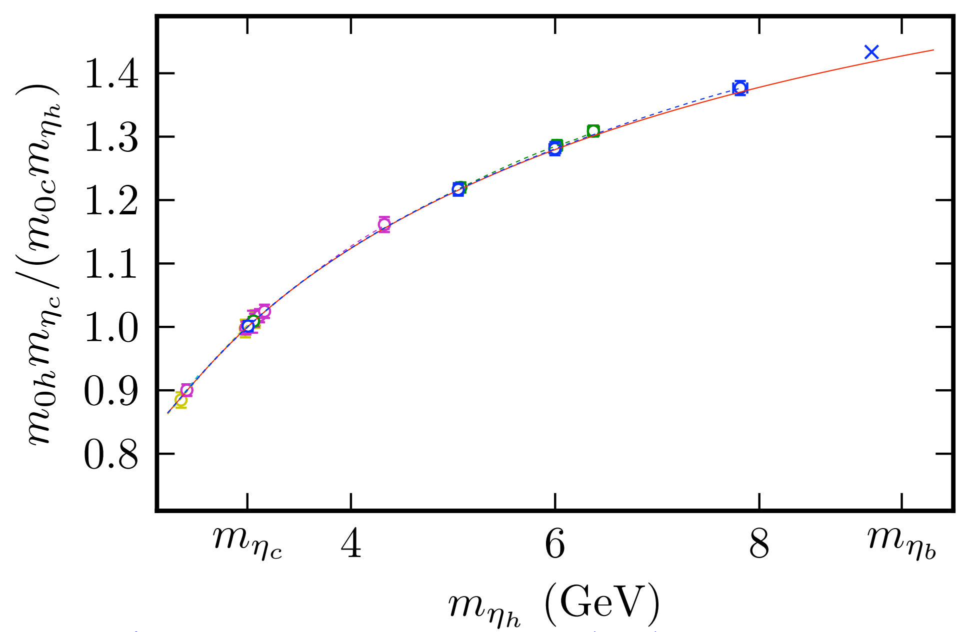
$$\left(\frac{m_{q1,latt}}{m_{q2,latt}}\right)_{a=0} = \frac{m_{q1,\overline{MS}}(\mu)}{m_{q2,\overline{MS}}(\mu)}$$

Not possible any other way ...

m_c/m_s *new* 2+1+1 with physical u.d:



m_b/m_c



$m_c/m_s = 11.652(65)$

$m_b/m_c = 4.51(4)$

$m_b/m_s = 52.90(44)$

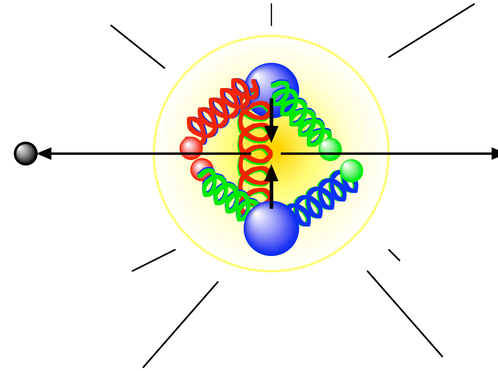
$\neq 3m_\tau/m_\mu$

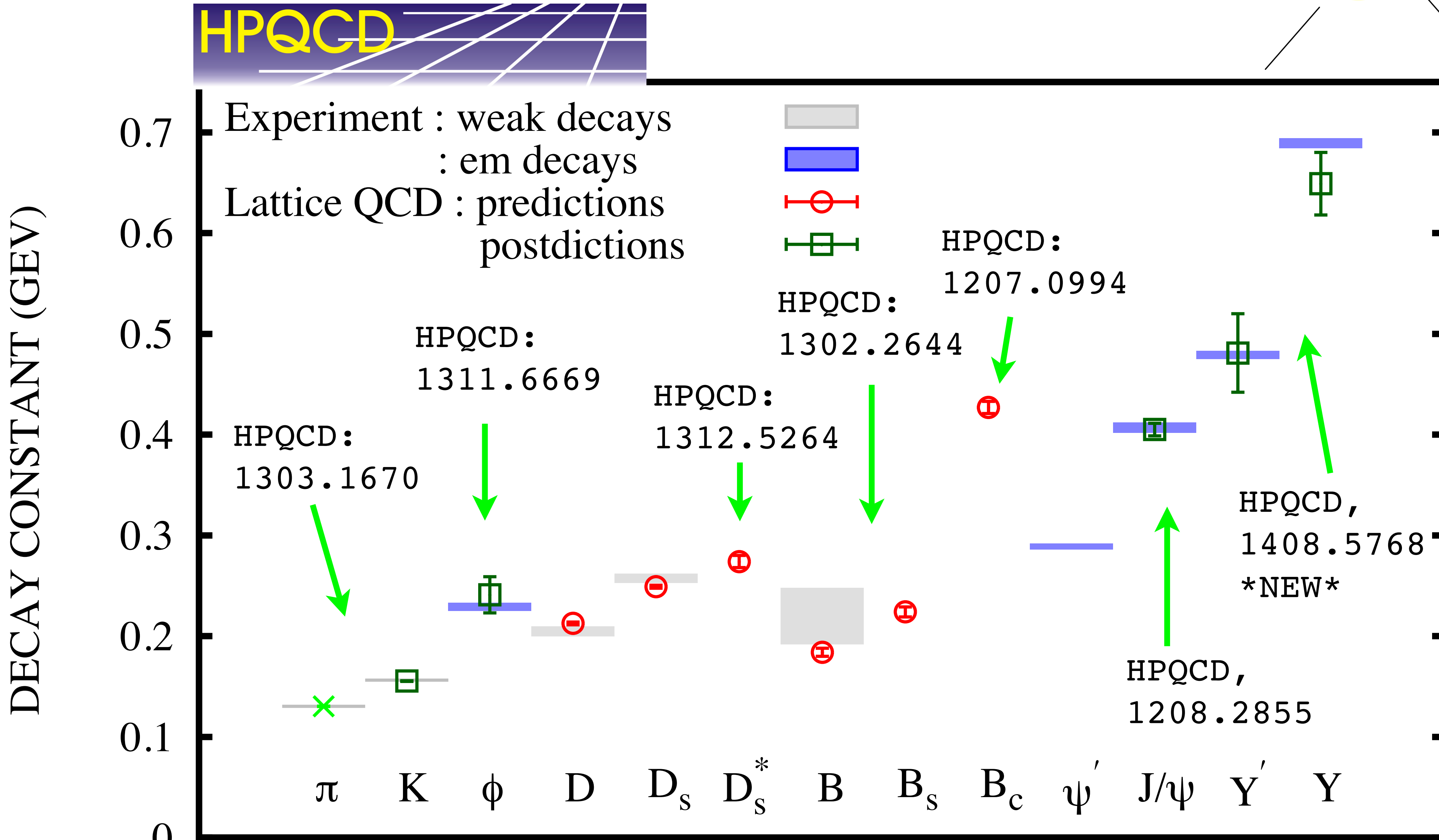
allows 1% accuracy in m_s (94.0(6) MeV)

Meson decay constants

Parameterises hadronic information needed for annihilation rate to W or photon:

$$\Gamma \propto f^2$$



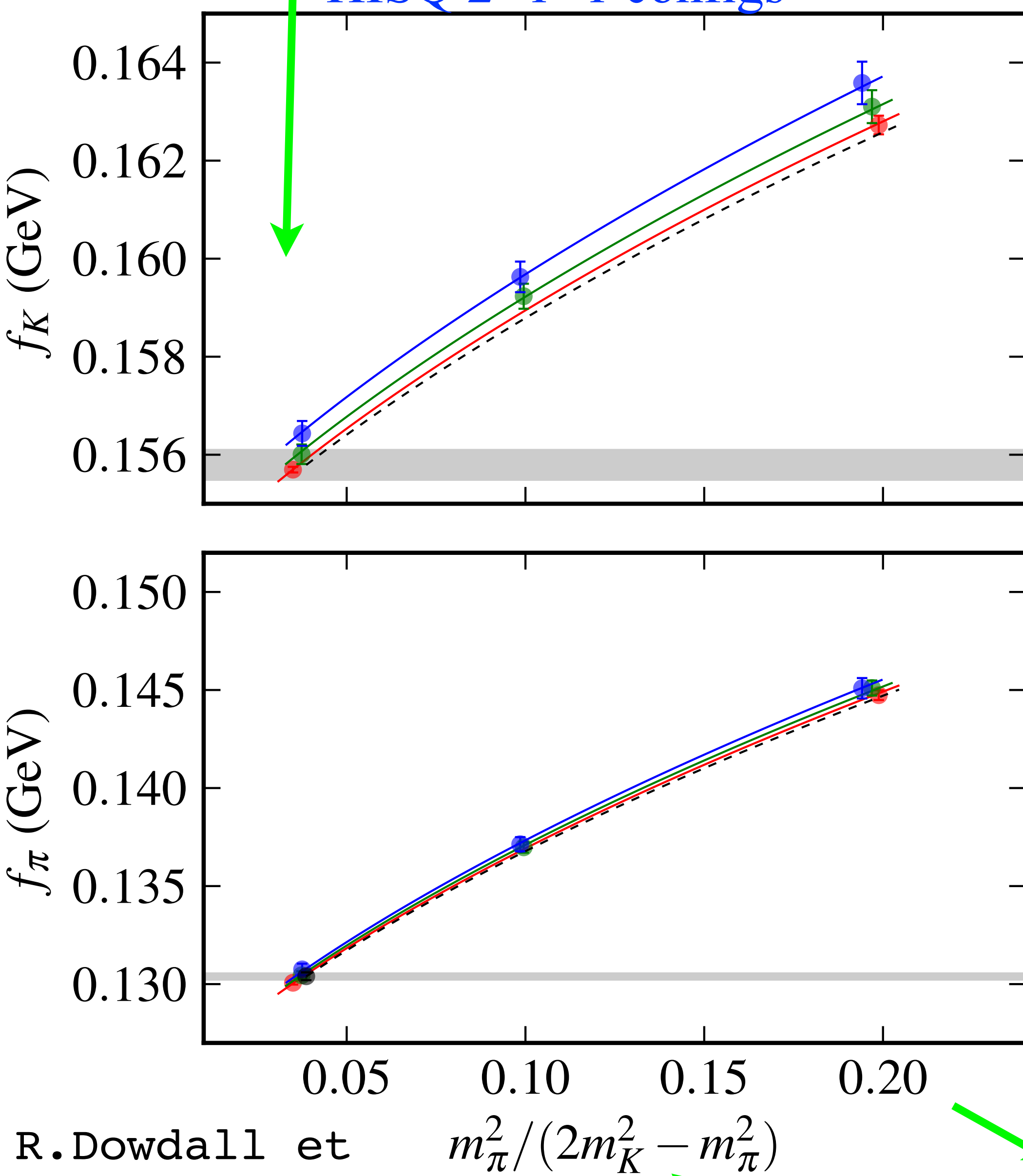


Aim for same ‘overview’ as for masses. Note different scale.

Constraining new physics with lattice QCD

* results at physical u/d quark masses*

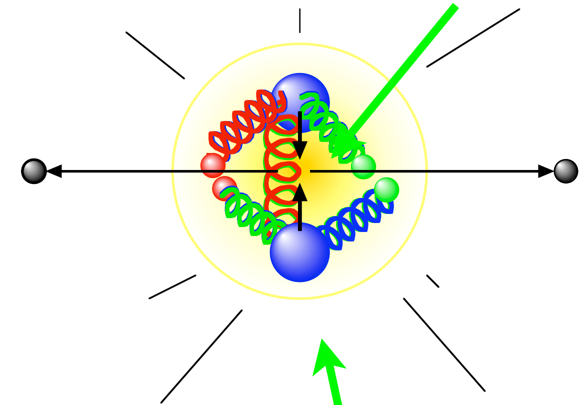
HISQ 2+1+1 configs



$= m_{u,d}/m_s$

R.Dowdall et al, HPQCD: 1303.1670

f_K/f_π $\quad V_{us}/V_{ud}$



Annihilation of K/π to W allows CKM element determination given decay constants from lattice QCD

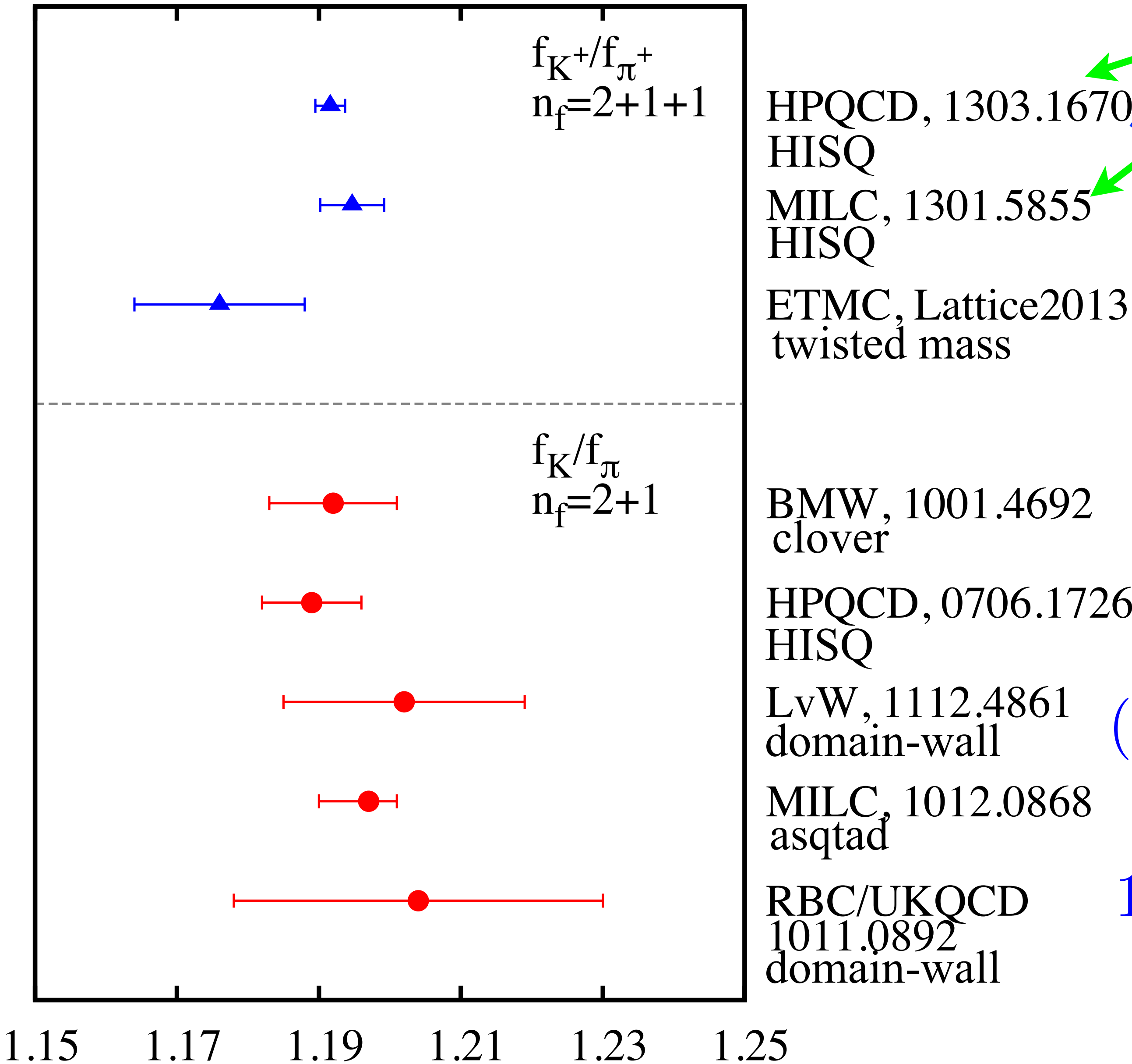
expt for $\dfrac{\Gamma(K^+ \to \ell\nu)}{\Gamma(\pi^+ \to \ell\nu)}$

$$\frac{|V_{us}|f_{K^+}}{|V_{ud}|f_{\pi^+}} = 0.27598(35)_{\mathrm{Br}(K^+)}(25)_{EM}$$

$\dfrac{f_{K^+}}{f_{\pi^+}}$ from lattice gives CKM

Comparison of results (note: $f_{K^+} < f_K$ by 0.3%)

good agreement from different formalisms



* results at physical u/d quark masses*

$$\frac{f_{K^+}}{f_{\pi^+}} = 1.1916(21)$$

$$\frac{|V_{us}|}{|V_{ud}|} = 0.23160(29)_{expt}(21)_{EM}(41)_{latt}$$

$$|V_{us}| = 0.22564$$

$$(28)_{\text{Br}}(20)_{EM}(40)_{latt}(5)_{V_{ud}}$$

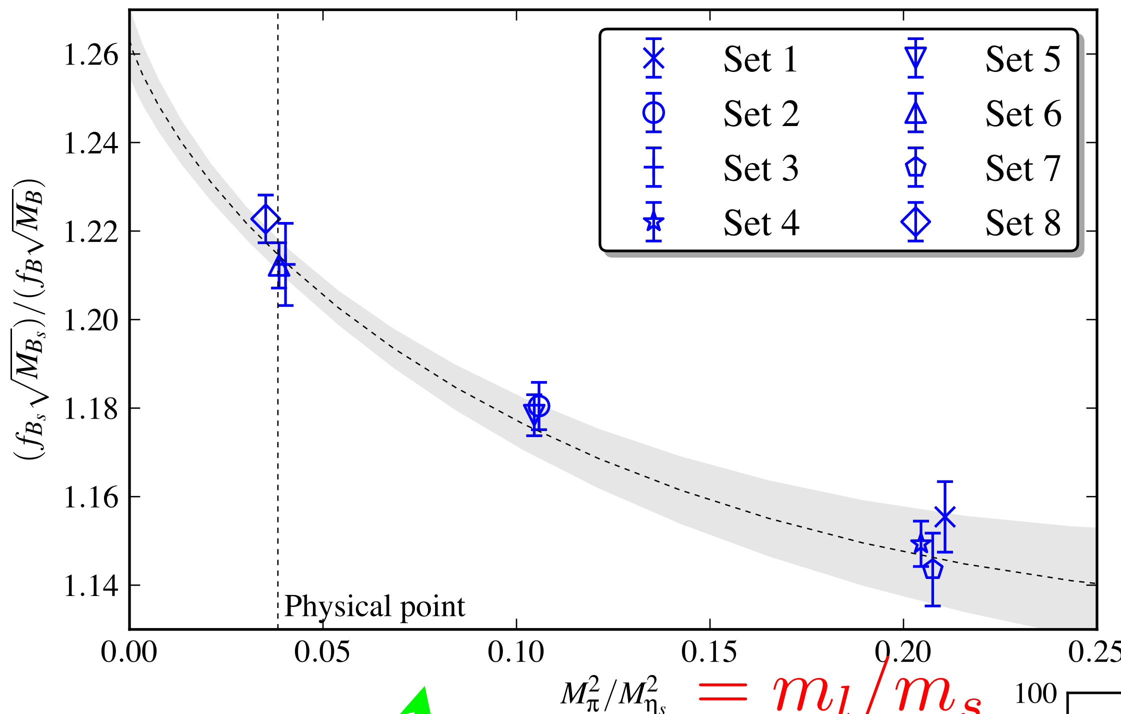
$$1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2$$

$$= -0.00009(51)$$

V_{ud} from nuclear β decay now needs improvement for unitarity test!

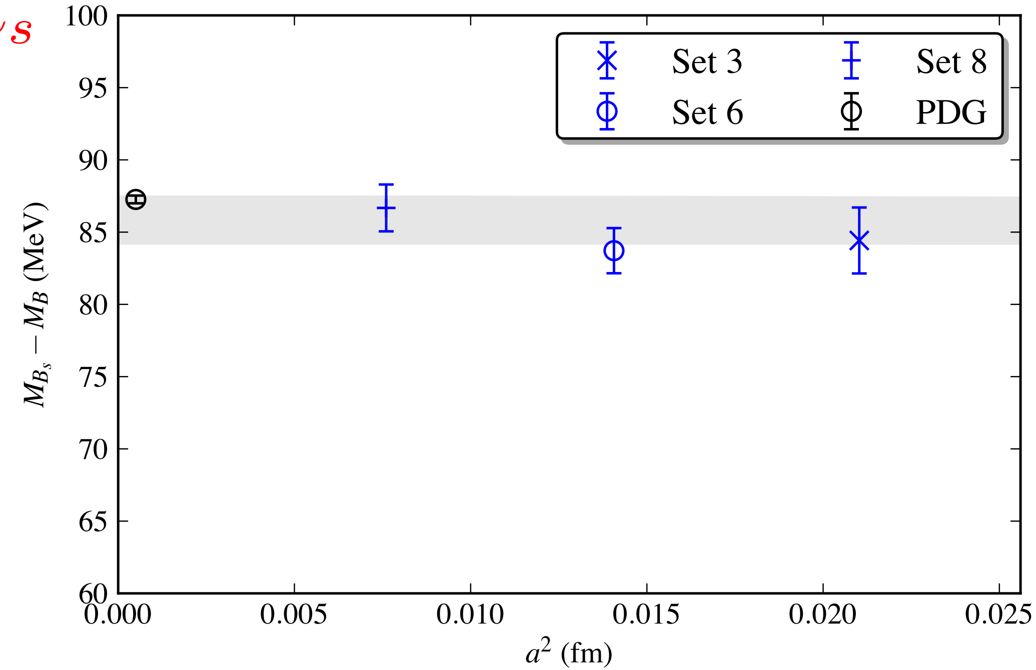
B meson decay constants: results from NRQCD b and physical u/d quarks

HPQCD: R Dowdall et al, 1302.2644.



MILC HISQ 2+1+1 configs with u/d down to physical values + improved NRQCD

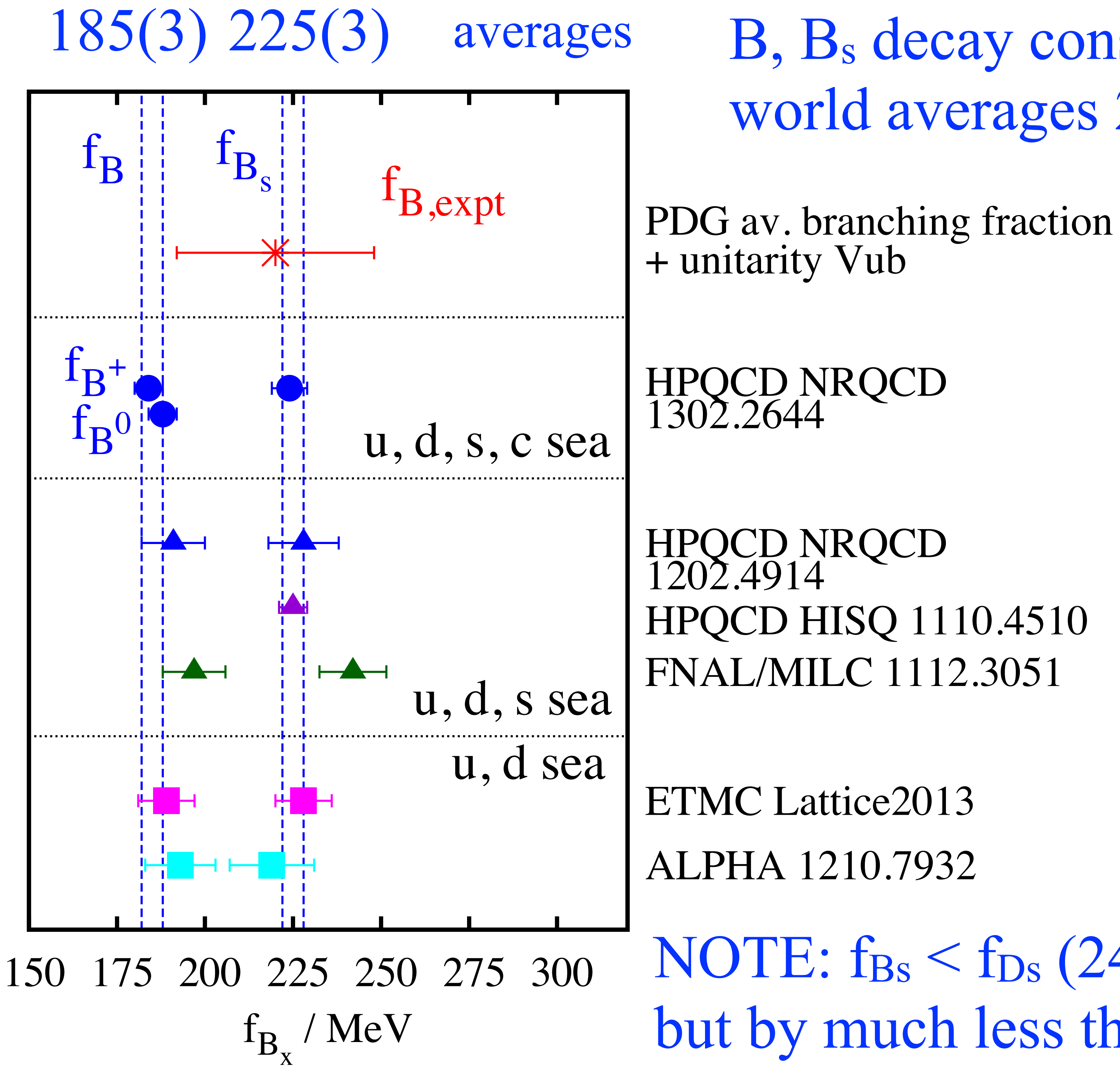
meson mass difference correct to 2%



B_s to B decay constant ratio accurate to 0.6% - since Z factors cancel.

Separate decay constants to 2%

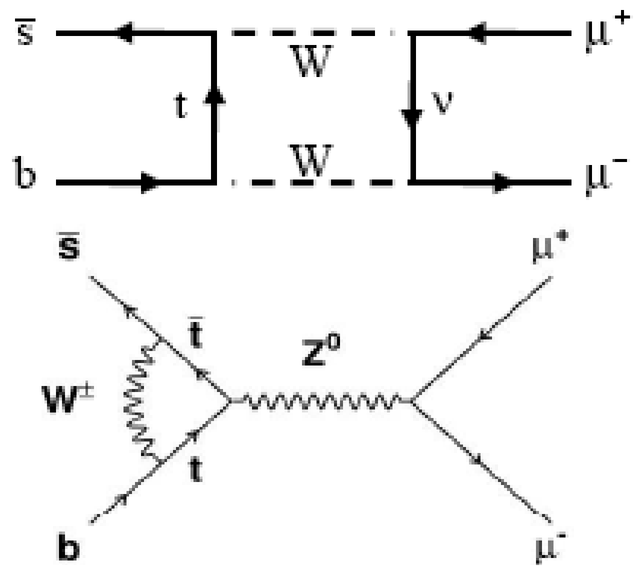
B, B_s decay constant world averages 2014



NOTE: $f_{Bs} < f_{Ds}$ (248 MeV) but by much less than LO HQET expects

Enables SM branching fraction to be determined for:

$$\mathrm{Br}(B_s \to \mu^+\mu^-) = A f_{B_s}^2 M_{B_s} |V_{tb}^* V_{ts}|^2 \tau(B_s)$$

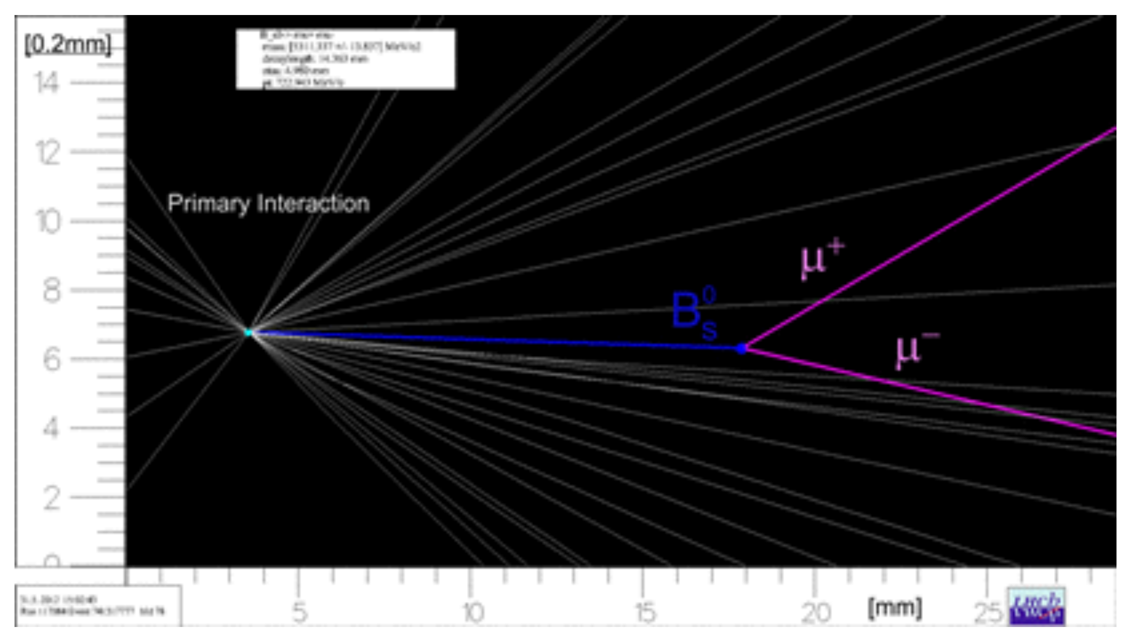


2013: Updated result from lattice QCD f_{Bs}: $3.47(19) \times 10^{-9}$

HPQCD: R Dowdall et al,1302.2644.

(including $\Delta\Gamma$ effect in time-integration)

LHCb: Nov. 2012



Combined CMS/LHCb:

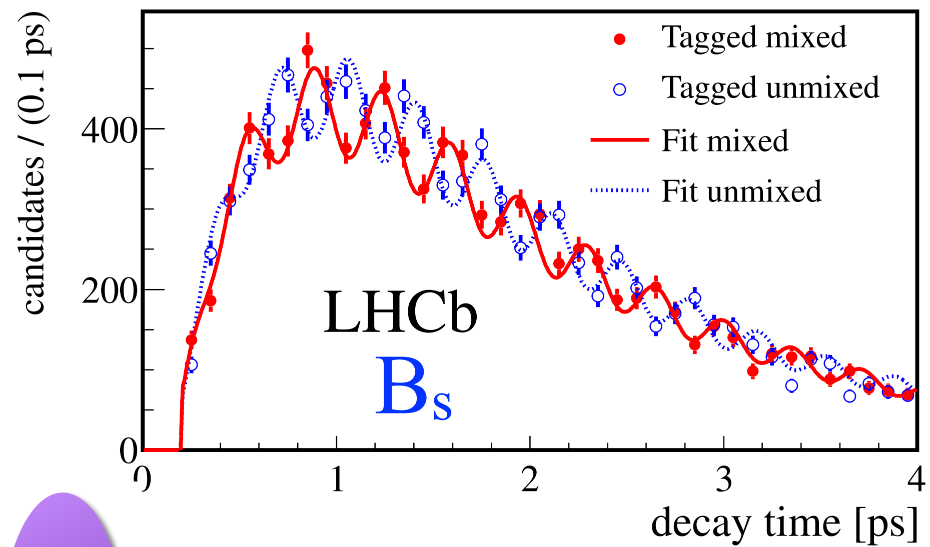
$\mathrm{Br} = 2.9(7) \times 10^{-9}$

Improved accuracy will allow strong test against SM.

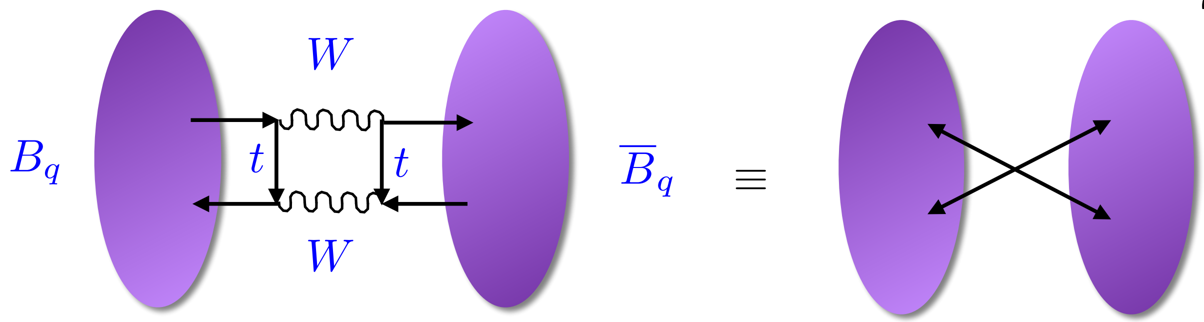
Determination of 4-quark matrix elements underway on 2+1+1 configs with physical u/d…

B/Bs mixing

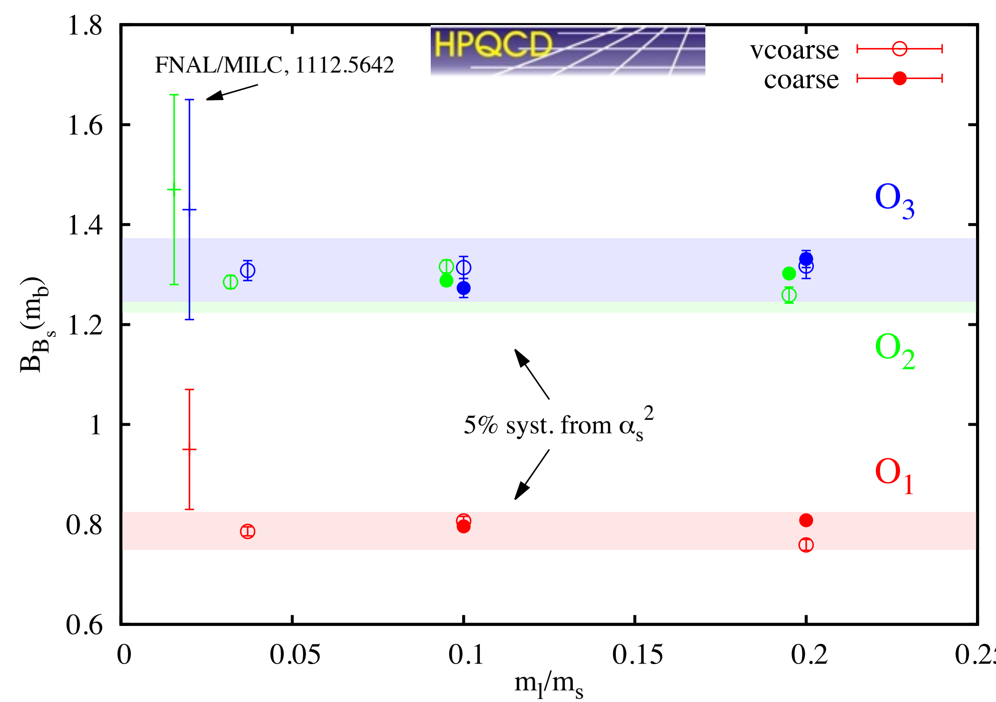
Neutral B flavour states 'oscillate' because they mix through SM loop effects - sensitive to BSM



LHCb,1304.4741



HPQCD, LAT2014



Rate depends on matrix elements of 4-quark operators and CKM V_{ts} and V_{td} .

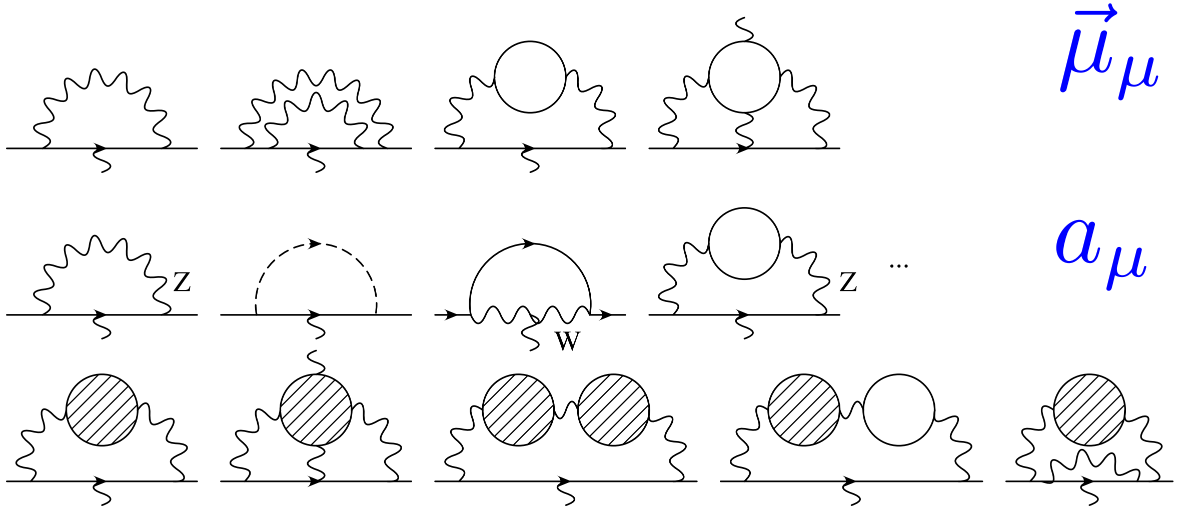
NEW Lattice QCD calculations inc. physical u/d quarks underway. Improved errors over previous results will be found ..

Anomalous magnetic moment of the muon

QED

EW

QCD



$$\vec{\mu}_\mu = g_\mu \frac{e}{2m}\vec{S}_\mu$$

$$a_\mu = (g-2)_\mu/2$$

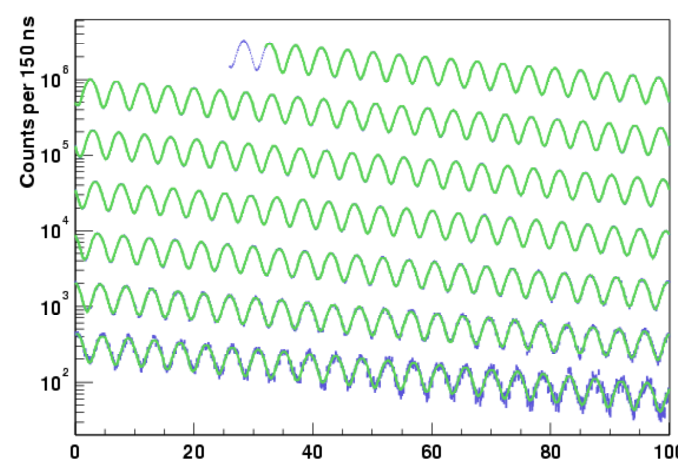
Blum et al, 1301.2607

sensitive to new physics ...

QED		$11658471.8845\,(9)(19)(7)(30) \times 10^{-10}$ $11658471.8951\,(9)(19)(7)(77) \times 10^{-10}$
EW		$15.4\,(2) \times 10^{-10}$
QCD	LO (e^+e^-)	$692.3\,(4.2) \times 10^{-10}$, $694.91\,(3.72)\,(2.10) \times 10^{-10}$
	LO (τ)	$701.5\,(4.7) \times 10^{-10}$
	HO HVP	$-9.79(9) \times 10^{-10}$
	HLbL	$10.5(2.6) \times 10^{-10}$

$R_{e^+e^-}$

BNL E821

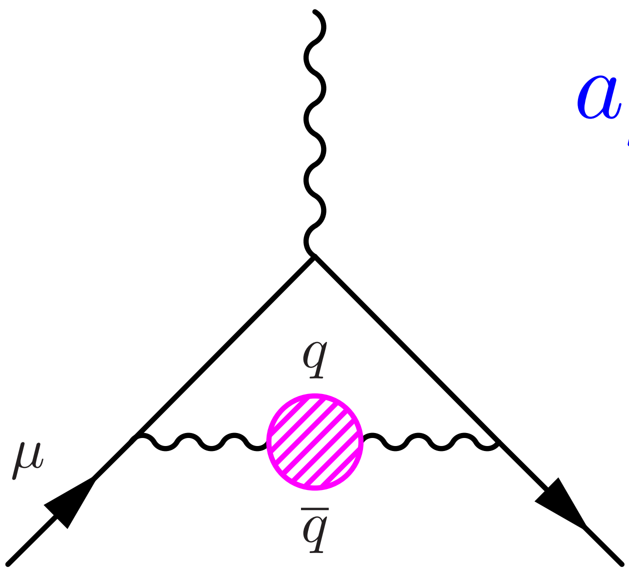


FNAL E989

Error in SM calc. dominated by that from hadronic vacuum polarisation - improve in lattice QCD?

Hadronic vacuum polarisation contribution to anomalous magnetic moment of muon $(g-2)_\mu/2$

B.Chakraborty et al, HPQCD: 1403.1778



a_μ differs between expt and the SM by $25(9)\times 10^{-10}$ *new physics*?

Uncertainty dominated by that from HVP contribution calculated from expt for $R_{e^+e^-}$

Can we improve ahead of E989 run?

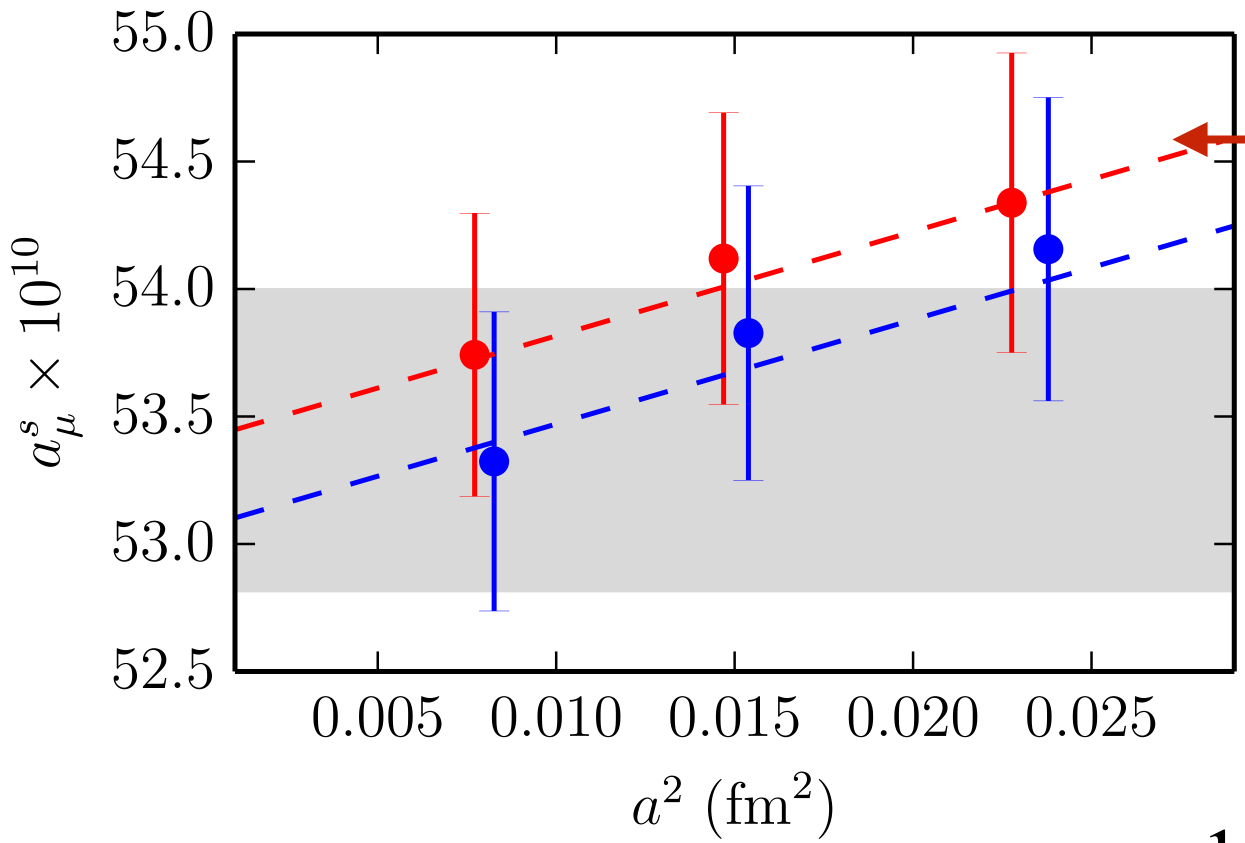
On lattice, calculate :

$$a^{(\mathrm{f})}_{\mu,\mathrm{HVP}} = \frac{\alpha}{\pi}\int_0^\infty dq^2 f(q^2)(4\pi\alpha Q_{\mathrm{f}}^2)\hat{\Pi}_{\mathrm{f}}(q^2)$$

very steep function, so small q^2 dominates

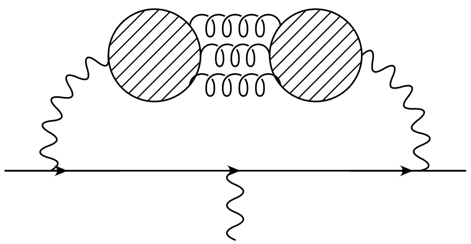
vacuum polarisation function

Determine the q^2 derivative moments of $\hat{\Pi}$ at $q^2=0$ from time moments of vector correlator and use Pade Approximants to evaluate the integral



s contrib. calculated on 2+1+1 with physical u/d quarks

still to do: "disconnected" pieces - expect very small



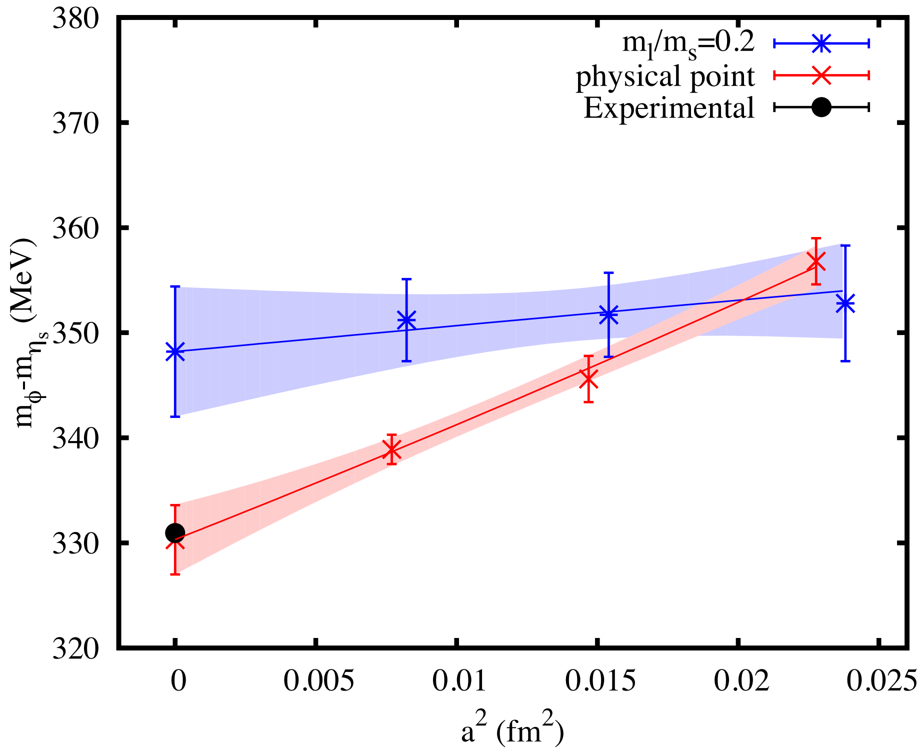
$$a_\mu^s = 53.41(59) \times 10^{-10}$$

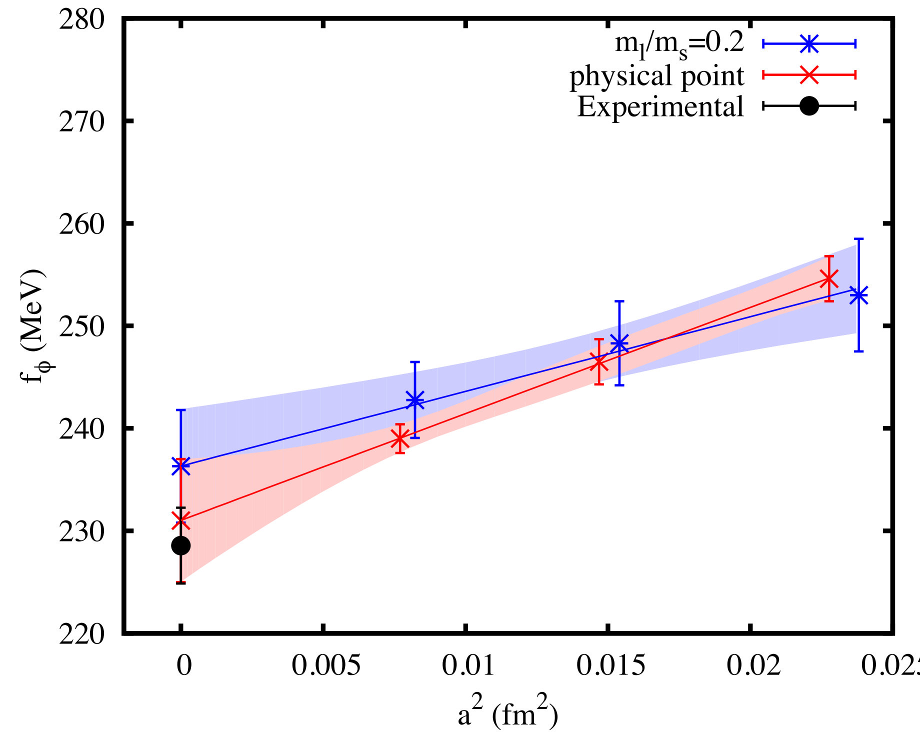
1% accurate

$$a_\mu^c = 14.42(39) \times 10^{-10}$$

2+1 results from earlier - agree well with $R_{e^+e^-}$

At large times vector correlator gives information about the ϕ meson - agrees well with expt for physical u/d





u/d calculation underway. Much noisier - currently getting $a_\mu^{HVP,LO} = 662(35) \times 10^{-10}$

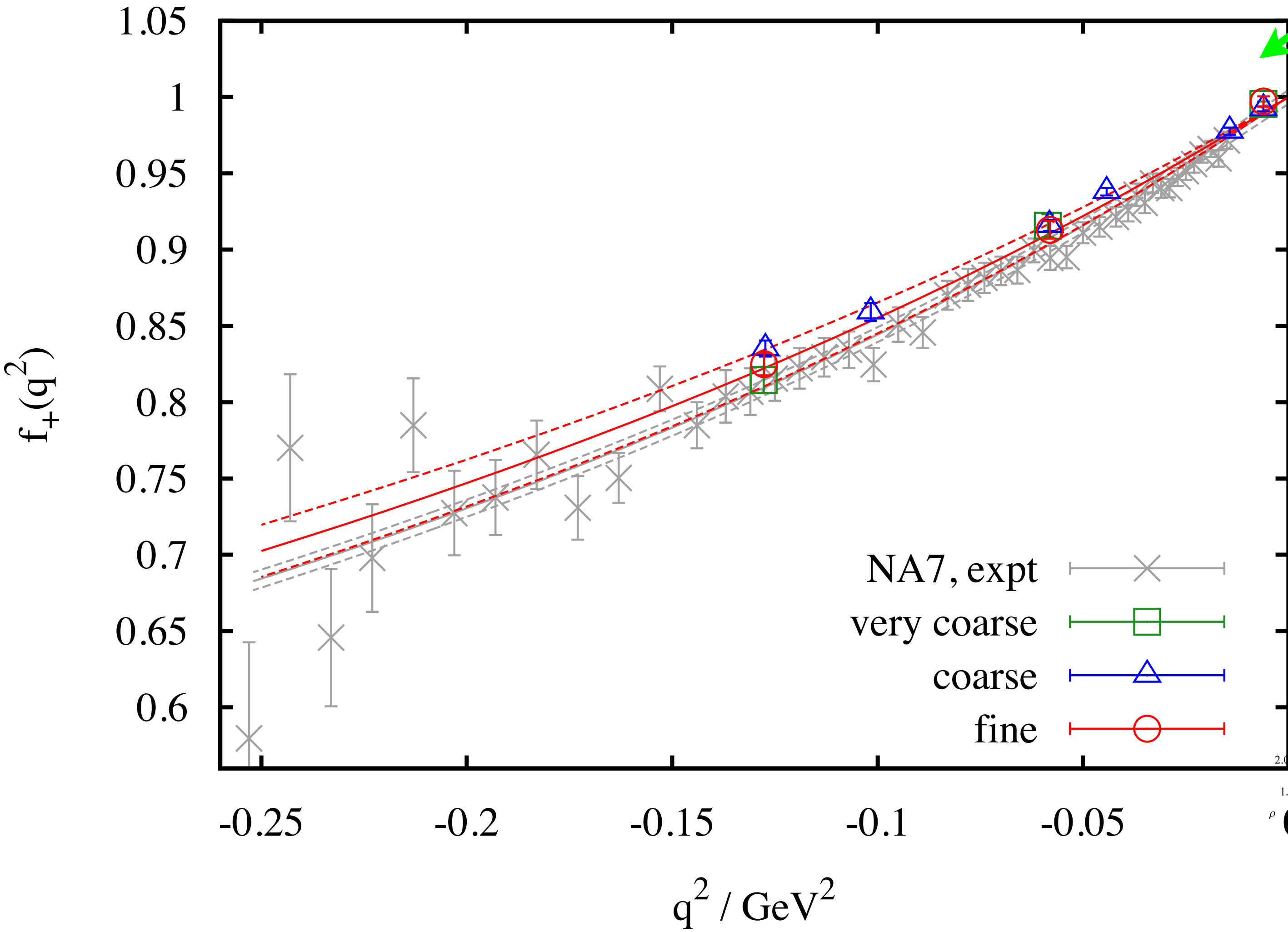
Expect to reduce error to 2-3% in current run.

Plan : 10x statistics in collaborn with MILC - should reduce errors below 1% by end 2015

Pion electromagnetic form factor

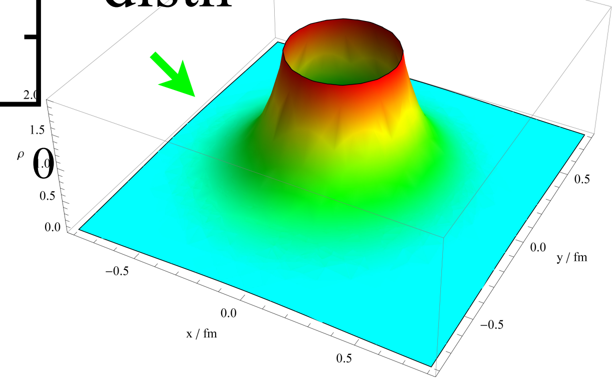
J. Koponen et al, HPQCD, 1311.3512

$\pi - e$ scattering probes π electric charge distribution



Working at physical u/d quark masses on HISQ 2+1+1 configurations. Agrees with experiment directly.

gives π charge distn



Scalar form factor in progress ….

www.physics.gla.ac.uk/HPQCD

Conclusion

• Lattice QCD results for gold-plated hadron masses and decay constants now providing stringent tests of QCD/SM, QCD parameters to 1% and input to BSM constraints.

Future

• Now working on '2nd generation' gluon configs with charm in the sea and $m_{u,d}$ at physical value. Will take a down below 0.05fm (so b quarks are 'light') and increase statistics by a factor of 10 on coarser lattices.

• Aim for 1% errors for B and B_s physics

• Improve noisier calculations such as muon g-2, calcs. inc 'disconnected diagrams', exotic hadrons etc.

• We need DiRAC III in 2015-16 to do this …

Spares

Look at error budgets to see how things will improve in future ...

1302.2644: calculation of B, B_s masses and decay constants

errors divided into extrapolation and other systematics:

Error %	Φ_{B_s}/Φ_B	$M_{B_s} - M_B$	Φ_{B_s}	Φ_B
EM:	0.0	1.2	0.0	0.0
a dependence:	0.01	0.9	0.7	0.7
chiral:	0.01	0.2	0.05	0.05
g:	0.01	0.1	0.0	0.0
stat/scale:	0.30	1.2	1.1	1.1
operator:	0.0	0.0	1.4	1.4
relativistic:	0.5	0.5	1.0	1.0
total:	0.6	2.0	2.0	2.1

for different quantities different systematics are important