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A spin analysis of the 4π channels produced in central pp interactions at 450 GeV/c

The WA102 Collaboration

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Abstract

The reactions $pp \to p_f(X^0)p_s$, where X^0 is observed decaying to $\pi^0\pi^0\pi^0\pi^0$, $\pi^+\pi^-\pi^+\pi^-$ and $\pi^+\pi^-\pi^0\pi^0$, have been studied at 450 GeV/c. There is evidence for an $a_2(1320)\pi$ decay mode of the $\eta_2(1645)$ and $\eta_2(1870)$ in the $\pi^+\pi^-\pi^0\pi^0$ and $\pi^+\pi^-\pi^+\pi^-$ final states. The $f_2(1950)$ is consistent with being a single resonance with a dominant $f_2(1270)\pi\pi$ decay mode. The $f_0(1370)$ is found to decay dominantly to $\rho\rho$ while the $f_0(1500)$ is found to decay to $\rho\rho$ and $\sigma\sigma$.

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The WA76, WA91 and WA102 collaborations have studied the centrally produced $\pi^+\pi^-\pi^+\pi^-$ final state in the reaction

$$pp \rightarrow p_f(\pi^+\pi^-\pi^+\pi^-)p_s \tag{1}$$

at 85 [1], 300 [2], and 450 GeV/c [3, 4]. The subscripts f and s indicate the fastest and slowest particles in the laboratory respectively. In addition to the $f_1(1285)$, which was observed at all energies, peaks were observed at 1.45 and 1.9 GeV in the 300 and 450 GeV/c data. In contrast, no clear evidence was seen for these states in the 85 GeV/c data of the same experiment [1]. The increased prominence of these states with increased incident energy [2] is consistent with the formation of these states via a double Pomeron exchange mechanism, which is predicted to be a source of gluonic states [5].

The peak at 1.9 GeV, called the $f_2(1950)$, was found to have $I^GJ^{PC} = 0^+2^{++}$ and decay to $f_2(1270)\pi\pi$ and $a_2(1320)\pi$ [3, 4]. It was not possible to determine whether this was one resonance with two decay modes, or two resonances. However, in a recent analysis of the centrally produced $\eta\pi^+\pi^-$ final state by the WA102 experiment [6], no evidence was found for a $J^{PC} = 2^{++}$ $a_2(1320)\pi$ decay mode. The peak at 1.45 GeV was shown to have $I^GJ^{PC} = 0^+0^{++}$ and decay to $\rho\rho$. It has been described as being due to the interference between the $f_0(1370)$ and the $f_0(1500)$ [3, 4] which implies that both states should have a substantial $\rho\rho$ decay mode. However, the initial analysis of the 4π channel in $p\overline{p}$ by Crystal Barrel [7] showed that the $f_0(1500)$ decays via $\sigma\sigma$ with a very weak coupling to $\rho\rho$. A recent preliminary analysis [8] of the Crystal Barrel data concludes that the $f_0(1370)$ decays strongly to $\sigma\sigma$ and $\rho\rho$ while the $f_0(1500)$ decays to $\sigma\sigma$ and $\pi^*\pi$ where the $\pi^*\pi$ decays to $\rho\pi$.

Therefore, there are still several unanswered questions on the decays of the $f_2(1950)$, $f_0(1370)$ and $f_0(1500)$. In order to address these questions, in this paper an analysis of several different 4π decay modes will be presented. In particular, to investigate the $\sigma\sigma$ contribution in the decays of the $f_0(1370)$ and $f_0(1500)$. One of the major problems of studying the $\pi^+\pi^-\pi^+\pi^-$ final state is the number of possible isobar decay modes that are present. Therefore a study will be made firstly of the $\pi^0\pi^0\pi^0\pi^0$ final state which has the nice feature that it is free from any contribution from $\rho(770)$, $a_1(1260)$ or $a_2(1320)$ isobars. Next an analysis will be presented of the $\pi^+\pi^-\pi^0\pi^0$ decay mode which has different systematic effects compared to the $\pi^+\pi^-\pi^+\pi^-$ decay. Finally a reanalysis of the $\pi^+\pi^-\pi^+\pi^-$ decay mode will be presented.

The data come from the WA102 experiment which has been performed using the CERN Omega Spectrometer, the layout of which is described in ref. [9]. The reaction

$$pp \rightarrow p_f(\pi^0\pi^0\pi^0\pi^0)p_s \tag{2}$$

has been studied at 450 GeV/c. Reaction (2) has been isolated from the sample of events having two outgoing charged tracks and eight γs reconstructed in the GAMS-4000 calorimeter, by first imposing the following cuts on the components of missing momentum: |missing P_x| < 14.0 GeV/c, |missing P_y| < 0.20 GeV/c and |missing P_z| < 0.16 GeV/c, where the x axis is along the beam direction. A correlation between pulse-height and momentum obtained from a system of scintillation counters was used to ensure that the slow particle was a proton.

Fig. 1a) shows the two photon mass spectrum for 8γ-events when the mass of the other three 2γ-pairs lies within a band around the π^0 mass (100–170 MeV). A clear π^0 signal is observed with a small background. Events belonging to reaction (2) have been selected using a kinematical fit (8C fit, four-momentum conservation being used and the masses of four π^0s being fixed). Events corresponding to the reaction $\eta\pi^0$, where the η decays to $3\pi^0$, have been removed by requiring $M(3\pi^0) \geq 0.6$ GeV. The major background to the $\pi^0\pi^0\pi^0\pi^0$ final state comes from the decay of the η' and $f_1(1285)$ to $\eta\pi^0\pi^0$ where the η decays to $3\pi^0$ and one π^0 is undetected. This produces events in the threshold to 1.3 GeV region of the $\pi^0\pi^0\pi^0\pi^0$ mass spectrum. A kinematical fit has been used to remove these events. Using real $\eta\pi\pi$ events a simulation has been performed to estimate the residual contamination in the $\pi^0\pi^0\pi^0\pi^0$ final state which is found to be less than 5 %.

The resulting $\pi^0\pi^0\pi^0\pi^0$ effective mass spectrum is shown in fig. 1b) and consists of 1438 events. There is a broad enhancement in the 2 GeV region, compatible with the $f_2(1950)$, but there is no clear peak in the 1.45 GeV region.

One of the aims of this paper is to investigate the role of the σ in the decays of the 4π final state. In order to determine the amount of $\sigma\sigma$ a parameterisation of the $\pi\pi$ S-wave (σ) is required. Parameters for the σ may be process dependent. The nature of the σ is not clearly established and parameterisations of it may be process dependent. It may be a resonant state or just two pions produced in relative S-wave. However, the parameterisation of the $\pi\pi$ S-wave is crucial in all these analyses in order to determine the amount of $\sigma\sigma$ observed. Assuming that the 1.3$-$1.5 GeV $\pi^0\pi^0\pi^0\pi^0$ mass region is dominated by the decay to $\sigma\sigma$, then in order to determine the best parameters for the σ, a study has been made of the $\pi^0\pi^0$ mass spectrum for the region $1.3 \leq \mathrm{M}(\pi^0\pi^0\pi^0\pi^0) \leq 1.5$ GeV, shown in fig. 1c). A Monte Carlo simulation has been performed using the decay $X^0 \to \sigma\sigma$ with $\sigma \to \pi^0\pi^0$. Formally, we describe the shape of the σ using a spin 0 relativistic Breit-Wigner amplitude of the form:

$$A(M_{\pi\pi}) = \left(\frac{q}{q_0}\right) \frac{\Gamma m_0}{m_0^2 - m_{\pi\pi}^2 - i m_0 \Gamma}$$

In order to describe the centrally produced mass spectrum the resulting function $|A(M_{\pi\pi})|^2$ has been multiplied by the kinematical factor $(M_{\pi\pi} - 4m_\pi^2)^{1/2}/M_{\pi\pi}^3$ [10]. The mass and width of the σ have been varied and the Monte Carlo data passed through the detector simulation and the resulting mass distributions were compared to the real data. The parameters that best describe the data are $m_0 = 450 \pm 50$ MeV and $\Gamma = 600 \pm 50$ MeV and the resulting mass spectrum (solid histogram) is superimposed on the real $\pi^0\pi^0$ mass spectrum shown in fig. 1c).

A spin-parity analysis of the $\pi^0\pi^0\pi^0\pi^0$ channel has been performed in 120 MeV $\pi^0\pi^0\pi^0\pi^0$ mass bins using an isobar model and the method described in ref. [3], assuming that only the $\sigma\sigma$ and $f_2(1270)\pi\pi$ intermediate states with $J^{PC} = 0^{++}$ or 2^{++} contribute. The data can be fitted using only the $J^{PC} = 0^{++}$ $\sigma\sigma$ wave and the $J^{PC} = 2^{++}$ $f_2(1270)\pi\pi$ wave with $J_Z = 0$ which are shown in figs. 1d) and e) respectively. Superimposed on fig. 1d) is the distribution that would be expected for the $f_0(1500)$; as can be seen there is little evidence for any $f_0(1370)$ contribution. Superimposed on fig. 1e) is what would be expected from the $f_2(1950)$; as can be seen it well describes the $J^{PC} = 2^{++}$ $f_2(1270)\pi\pi$ wave, showing that the peak at 1.95 GeV is consistent with being a single resonance.

An analysis has next been performed on the centrally produced $\pi^+\pi^-\pi^0\pi^0$ system. The reaction

$$pp \to p_f(\pi^+\pi^-\pi^0\pi^0)p_s \tag{3}$$

has been isolated from the sample of events having four outgoing charged tracks and four γs reconstructed in the GAMS-4000 calorimeter, by first imposing the following cuts on the components of the missing momentum: |missing P_x| < 17.0 GeV/c, |missing P_y| < 0.16 GeV/c and |missing P_z| < 0.12 GeV/c. The two photon mass spectrum (not shown) when the mass of the other 2γ-pair lies within a band around the π^0 mass (100–170 MeV) shows a clear π^0 signal with small background. Events belonging to reaction (3) have been selected using a kinematical fit (6C fit, four-momentum conservation being used and the masses of two π^0s being fixed). Events containing a fast $\Delta^{++}(1232)$ were removed if $M(p_f\pi^+) < 1.3$ GeV, which left 283 408 centrally produced $\pi^+\pi^-\pi^0\pi^0$ events.

Fig. 2a shows the $\pi^+\pi^-\pi^0\pi^0$ effective mass spectrum. The mass spectrum is very similar to that observed previously in the $\pi^+\pi^-\pi^+\pi^-$ channel [4], namely, a clear peak at 1.28 GeV associated with the $f_1(1285)$, a peak at 1.45 GeV and a broad enhancement around 2.0 GeV.

A spin-parity analysis of the $\pi^+\pi^-\pi^0\pi^0$ channel has been performed using an isobar model and the method described in ref. [3]. Assuming that only angular momenta up to 2 contribute, the intermediate states considered are

$\sigma\sigma$,	$\sigma\rho^0$,		
$\rho^+\rho^-$,	$\rho^\pm(\pi^\mp\pi^0)_{Swave}$,	$\rho^\pm(\pi^\mp\pi^0)_{Pwave}$,	$\rho^\pm(\pi^\mp\pi^0)_{Dwave}$,
$a_1(1260)\pi$,	$a_2(1320)\pi$,	$f_2(1270)\sigma$,	$f_2(1270)(\pi\pi)_{Swave}$,
$f_2(1270)\rho^0$,	$f_2(1270)f_2(1270)$		
$\pi^*(1300)\pi$			

Three parameterisations of the σ have been tried, that of Au, Morgan and Pennington [10], that of Zou and Bugg [11] and the parameterisation found to fit the σ contribution in the $\pi^0\pi^0\pi^0\pi^0$ channel described above. Three different parameterisations of the $\pi^*(1300)$ have been tried. In the Crystal Barrel analysis two parameterisations of the $\pi^*(1300)$ have been found, one with M = 1114 MeV, Γ = 340 MeV [7] and a second with M = 1400 MeV, Γ = 275 MeV [8]. The third parameterisation uses the PDG values [12].

Different combinations of waves and isobars have been tried and insignificant contributions have been removed from the final fit. The best fit is shown in fig. 2 and consists of the following waves: $J^{PC} = 1^{++}$ $\rho\rho$ with $|J_Z| = 1$ fig. 2b), $J^{PC} = 0^{++}$ $\rho\rho$ fig. 2c), $J^{PC} = 0^{++}$ $\sigma\sigma$ fig. 2d), $J^{PC} = 2^{-+}$ $a_2(1320)\pi$ with $|J_Z| = 1$ fig. 2e) and $J^{PC} = 2^{++}$ $f_2(1270)\pi\pi$ with $J_Z = 0$ fig. 2f). As in the case of the analysis of the $\eta\pi^+\pi^-$ channel [6] there is no need for any $J^{PC} = 2^{++}$ $a_2(1320)\pi$ wave. The $J^{PC} = 0^{++}$ $\sigma\sigma$ wave is only required if the parameterisation of the σ found from the $\pi^0\pi^0\pi^0\pi^0$ analysis is used. If the parameterisations of the σ used to fit the $\pi^+\pi^-\pi^+\pi^-$ channel in our previous publication [4] are used here, the same conclusion would be drawn, i.e. no $J^{PC} = 0^{++}$ $\sigma\sigma$ wave is required. Hence as was stated in the introduction the parameterisation used to describe the σ is crucial. There is no need for any $J^{PC} = 0^{++}$ $\pi^*(1300)\pi$ wave irrespective of the parameterisation used.

Superimposed on the $J^P = 1^+$ $\rho\rho$ wave shown in fig. 2b) is a Breit Wigner convoluted with a Gaussian used to describe the $f_1(1285)$ in the fit to the $\pi^+\pi^-\pi^+\pi^-$ mass spectrum [4]. As can be seen the $f_1(1285)$ is well described.

The $J^P = 0^+\rho\rho$ distribution in fig. 2c) shows a peak at 1.45 GeV together with a broad enhancement around 2 GeV. A fit has been performed to the $J^P = 0^+\rho\rho$ amplitude in fig. 2c) using a single channel K matrix formalism [13] including poles to describe the interference between the $f_0(1370)$, the $f_0(1500)$ and a possible state at 2 GeV. No account has been made for the $\rho\rho$ threshold in this fit. The result of the fit is superimposed on the $J^P = 0^+\rho\rho$ distribution shown in fig. 2c) and well describes the data. The resulting T-matrix sheet II pole positions [14] for the resonances are

$$\begin{array}{llll} f_0(1370) & \mathrm{M} = (1309 \pm 24) & -i\,(163 \pm 26) & \mathrm{MeV} \\ f_0(1500) & \mathrm{M} = (1513 \pm 12) & -i\,(\ 58 \pm 12) & \mathrm{MeV} \\ f_0(2000) & \mathrm{M} = (1989 \pm 22) & -i\,(\ 224 \pm 42) & \mathrm{MeV} \end{array}$$

These parameters are consistent with the PDG [12] values for the $f_0(1370)$ and $f_0(1500)$.

The $J^P = 0^+\sigma\sigma$ distribution in fig. 2d) shows a peak at 1.5 GeV. Superimposed on fig. 2d) is what would be expected from the $f_0(1500)$; as can be seen there is little evidence for any $f_0(1370)$ contribution. A result similar to what was found in the analysis of the $\pi^0\pi^0\pi^0\pi^0$ final state. Correcting for the unseen $\rho\rho$ and $\sigma\sigma$ decay modes the branching ratio of $f_0(1500)$ to $\rho\rho/\sigma\sigma = 3.3 \pm 0.5$.

These results disagree with the values found by the Crystal Barrel collaboration [7, 8] which found a considerably larger contribution of $\sigma\sigma$ in the decays of the $f_0(1370)$ and $f_0(1500)$. In order to determine the maximum $\sigma\sigma$ contribution the parameters of the σ would need to be changed such that it has a similar shape to the ρ. In this case the difference between a $\rho\rho$ and $\sigma\sigma$ decay mode would be determined by the relative angular momentum between the pions from the decay of the ρ or σ. Using such a parameterisation for the σ we can determine a maximum contribution in the decay of the $f_0(1370)$ to $\sigma\sigma$ of 30 % of the $\rho\rho$ decay. For the $f_0(1500)$ such a parameterisation of the σ would yield equal decay rates to $\sigma\sigma$ and $\rho\rho$.

As in the case of the $\eta\pi^+\pi^-$ channel [6], the $J^{PC} = 2^{-+}$ $a_2(1320)\pi$ wave is consistent with being due to two resonances, the $\eta_2(1645)$ and the $\eta_2(1870)$. Superimposed on fig. 2e) is the result of a fit using two resonances to describe the $\eta_2(1645)$ and $\eta_2(1870)$. The masses and widths determined for each resonance are given in table 1 and are consistent with those found previously in the $\eta\pi\pi$ final state [6].

Fig. 2f) shows the $J^P = 2^+$ $f_2(1270)(\pi\pi)_{Swave}$ distribution which has been fitted with a single Breit-Wigner, with M = 1980 $\pm$ 22 MeV and Γ = 520 $\pm$ 50 MeV. These values are compatible with those coming from the fit to $f_2(1950)$ from the $\pi^+\pi^-\pi^+\pi^-$ mass spectrum [4].

Finally, a reanalysis of the $\pi^+\pi^-\pi^+\pi^-$ channel has now been performed using the new parameterisation of the σ described above. A new version of the Monte Carlo program has also been used which has been improved since the previous publication [4]. The $\pi^+\pi^-\pi^+\pi^-$ effective mass spectrum is shown in fig. 3a) and consists of 1 167 089 centrally produced events.

The analysis has been performed independently from the $\pi^+\pi^-\pi^0\pi^0$ analysis and different combinations of waves and isobars have been tried and insignificant contributions have been removed from the final fit. As in the original analysis [4] of the $\pi^+\pi^-\pi^+\pi^-$ system the addition of the $J^{PC} = 2^{++}$ $a_2(1320)\pi$ wave improved the Log Likelihood in the 1.8 to 1.9 GeV mass interval and has the effect of splitting the $f_2(1950)$ signal. If we use the information from the $\eta\pi^+\pi^-$ [6] and $\pi^+\pi^-\pi^0\pi^0$ analysis and reject this wave then the best fit is shown in fig. 3 and consists of the following waves: $J^{PC} = 1^{++}$ $\rho\rho$ with $|J_Z| = 1$ fig. 3b), $J^{PC} = 0^{++}$ $\rho\rho$ fig. 3c), $J^{PC} = 0^{++}$ $\sigma\sigma$ fig. 3d), $J^{PC} = 2^{-+}$ $a_2(1320)\pi$ with $|J_Z| = 1$ fig. 3e) and $J^{PC} = 2^{++}$ $f_2(1270)\pi\pi$ with $J_Z = 0$ fig. 3f). The results are then very similar to those found for the $\pi^+\pi^-\pi^0\pi^0$ channel.

A fit to the $J^P = 2^+$ $f_2(1270)(\pi\pi)_{Swave}$ distribution shown in fig. 3f) gives M = 1940 $\pm$ 22 MeV and $\Gamma = 485 \pm 55$ MeV. In order to get further confidence that the $f_2(1950)$ has a dominant $f_2(1270)\pi\pi$ decay mode a measurement of its rate to $\pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-\pi^0\pi^0$ and $\pi^0\pi^0\pi^0\pi^0$ has been made. The predicted ratio for an I = 0 state would be $\pi^+\pi^-\pi^+\pi^-$:$\pi^+\pi^-\pi^0\pi^0$:$\pi^0\pi^0\pi^0\pi^0$ = 4 : 4 : 1. The measured rate is 4 : 3.7 $\pm$ 0.2 : 0.95$\pm$0.1 consistent with the prediction. However, if the $a_2(1320)\pi$ wave was introduced in the $\pi^+\pi^-\pi^+\pi^-$ analysis the measured $\pi^+\pi^-\pi^+\pi^-$ rate would drop by a factor of 2.

Therefore, although the Log Likelihood would indicate the presence of a $J^{PC} = 2^{++}$ $a_2(1320)\pi$ wave in the fit of the $\pi^+\pi^-\pi^+\pi^-$ channel, the lack of it in the $\eta\pi^+\pi^-$ and $\pi^+\pi^-\pi^0\pi^0$ together with the fact that the measured decay rates would be wrong indicates how dangerous it can be to rely on Log Likelihood from a single channel alone to determine the existence of a decay mode.

Superimposed on the $J^P = 1^+$ $\rho\rho$ wave shown in fig. 3b) is a Breit Wigner convoluted with a Gaussian used to describe the $f_1(1285)$ in the fit to the mass spectrum. As can be seen the $f_1(1285)$ is well described.

A fit to the $J^P = 0^+\rho\rho$ distribution shown in fig. 3c) has been performed using the parametrisation used in the $\pi^+\pi^-\pi^0\pi^0$ channel. The T-matrix sheet II pole positions for the resonances are

$f_0(1370)$	M	$= (1295 \pm 24)$	$-i\,(169 \pm 26)$	MeV
$f_0(1500)$	M	$= (1509 \pm 12)$	$-i\,(44 \pm 12)$	MeV
$f_0(2000)$	M	$= (1995 \pm 22)$	$-i\,(218 \pm 42)$	MeV

These parameters are in good agreement with those found in the $\pi^+\pi^-\pi^0\pi^0$ final state.

The $J^P = 0^+\sigma\sigma$ distribution in fig. 3d) shows a peak at 1.5 GeV. Superimposed on fig. 3d) is what would be expected from the $f_0(1500)$. Similar to what was found in the $\pi^0\pi^0\pi^0\pi^0$ and $\pi^+\pi^-\pi^0\pi^0$ final states there is little evidence for any $f_0(1370)$ contribution. Correcting for the unseen $\rho\rho$ and $\sigma\sigma$ decay modes the branching ratio of $f_0(1500)$ to $\rho\rho/\sigma\sigma = 2.6 \pm 0.4$ which is consistent with the value found from the $\pi^+\pi^-\pi^0\pi^0$ final state. As in the $\pi^+\pi^-\pi^0\pi^0$ channel a parameterisation of the σ has been used with a shape similar to that of the $\rho(770)$ which give an upper limit to the $\sigma\sigma$ contribution in the decays of the $f_0(1370)$ and $f_0(1500)$. For the $f_0(1370)$ the maximum contribution is 25 % of the $\rho\rho$ decay. For the $f_0(1500)$ the maximum $\sigma\sigma$ contribution is 80 % of the $\rho\rho$ decay.

Fig. 3e) shows the $J^{PC} = 2^{-+}$ $a_2(1320)\pi$ wave. Superimposed is the result of a fit using two resonances to describe the $\eta_2(1645)$ and $\eta_2(1870)$. The masses and widths determined for each resonance are given in table 1 and as can be seen are consistent with those found for the $\eta\pi\pi$ [6] and $\pi^+\pi^-\pi^0\pi^0$ final states. Table 1 also gives the combined masses and widths from all the decay modes, including those found in the $\eta\pi^+\pi^-$ channel [6], where the common systematic errors have been taken into account. In both the $\pi^+\pi^-\pi^0\pi^0$ and $\pi^+\pi^-\pi^+\pi^-$ channels there is also evidence for a $J^P = 2^+$ $\rho\rho$ wave. Except for evidence for the $f_2(1270)$ this wave is a broad, structure-less distribution.

In order to gain further confidence in the results, we have measured the rate to $\pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-\pi^0\pi^0$ and $\pi^0\pi^0\pi^0\pi^0$ for the other waves observed. As can be seen from table 2 the results obtained are consistent with the expectations.

In the analyses presented in this paper, the decay modes of the $f_0(1370)$ and $f_0(1500)$ are in disagreement with what has been found by the Crystal Barrel collaboration. In order to demonstrate the clear need for a sizable $\rho\rho$ contribution to the peak at 1.45 GeV a study has been made of the 2π and 3π effective mass spectra for the mass range $1.4 \leq \mathrm{M}(4\pi) \leq 1.5$ GeV and for $dP_T \leq 0.2$ GeV for which the peak at 1.45 GeV is most prominent [4]. Fig. 4a) and b) show the $\pi^+\pi^-$ and $\pi^+\pi^-\pi^\pm$ mass spectra respectively from this region for the $\pi^+\pi^-\pi^+\pi^-$ final state. Superimposed on the mass spectra as a dashed line is what would be expected from a $\rho\rho$ final state and shaded is the contribution from a $\sigma\sigma$ final state using the parametrisation of the σ found from the $\pi^0\pi^0\pi^0\pi^0$ channel. The solid curve shows the sum of the two contributions and as can be seen, apart from an excess of events in the K^0 mass region of the 2π mass spectrum, the simulation well describes the experimental $\pi^+\pi^-$ and $\pi^+\pi^-\pi^\pm$ mass spectra. Therefore it can be seen that a strong $\rho\rho$ contribution is needed in this mass region and there is no need for any $\pi^*(1300)$ contribution to the 3π mass spectrum. A similar conclusion is found from an analysis for the $\pi^+\pi^-\pi^0\pi^0$ final state.

In summary, there is evidence for an $a_2(1320)\pi$ decay mode of the $\eta_2(1645)$ and $\eta_2(1870)$ in the $\pi^+\pi^-\pi^0\pi^0$ and $\pi^+\pi^-\pi^+\pi^-$ final states. The $f_2(1950)$ is consistent with being a single resonance with a dominant $f_2(1270)\pi\pi$ decay mode. The $f_0(1370)$ is found to decay dominantly to $\rho\rho$ while the $f_0(1500)$ is found to decay to $\rho\rho$ and $\sigma\sigma$.

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Tables

Table 1: Parameters of the $\eta_2(1645)$ and $\eta_2(1870)$

Resonance	Final state	Mass (MeV)	Width (MeV)
$\eta_2(1645)$	$\pi^+\pi^-\pi^0\pi^0$	1621 ± 11	168 ± 23
$\eta_2(1645)$	$\pi^+\pi^-\pi^+\pi^-$	1624 ± 12	173 ± 22
$\eta_2(1645)$	Combined	1617 ± 8	177 ± 18
$\eta_2(1870)$	$\pi^+\pi^-\pi^0\pi^0$	1862 ± 19	222 ± 28
$\eta_2(1870)$	$\pi^+\pi^-\pi^+\pi^-$	1843 ± 23	226 ± 32
$\eta_2(1870)$	Combined	1844 ± 13	228 ± 23

Table 2: The predicted rate for I = 0 and measured rate to $\pi^+\pi^-\pi^+\pi^-$: $\pi^+\pi^-\pi^0\pi^0$: $\pi^0\pi^0\pi^0\pi^0$ for different waves.

Wave	Predicted ratio	Measured ratio
2^{++} $f_2(1270)\pi\pi$	4 : 4 : 1	4 : 3.7±0.2 : 0.95±0.2
1^{++} $\rho\rho$	1 : 2 : 0	1 : 2.1±0.2 : 0.
0^{++} $\rho\rho$	1 : 2 : 0	1 : 1.9±0.2 : 0.
2^{-+} $a_2(1320)\pi$	1 : 2 : 0	1 : 2.0±0.2 : 0.
0^{++} $\sigma\sigma$	4 : 4 : 1	4 : 3.9±0.2 : 1.2±0.3

Figures

Figure 1: The $\pi^0\pi^0\pi^0\pi^0$ channel. a) M($\gamma\gamma$) when the other three $\gamma\gamma$ pairs lie in the π^0 region, b) M($\pi^0\pi^0\pi^0\pi^0$) and c) M($\pi^0\pi^0$) for $1.3 \leq$ M($\pi^0\pi^0\pi^0\pi^0$) ≤ 1.5 GeV. Results of the spin analysis d) the $J^{PC} = 0^{++}$ $\sigma\sigma$ wave and e) the $J^{PC} = 2^{++}$ $f_2(1270)\pi\pi$ wave with fits described in the text.

Figure 2: The $\pi^+\pi^-\pi^0\pi^0$ channel. a) The total mass spectrum, b) 1^{++} $\rho\rho$, c) 0^{++} $\rho\rho$, d) 0^{++} $\sigma\sigma$, e) 2^{-+} $a_2(1320)\pi$ and f) 2^{++} $f_2(1270)\pi\pi$. The superimposed curves are the resonance contributions coming from the fits described in the text.

Figure 3: The $\pi^+\pi^-\pi^+\pi^-$ channel. a) The total mass spectrum, b) 1^{++} $\rho\rho$, c) 0^{++} $\rho\rho$, d) 0^{++} $\sigma\sigma$, e) 2^{-+} $a_2(1320)\pi$ and f) 2^{++} $f_2(1270)\pi\pi$. The superimposed curves are the resonance contributions coming from the fits described in the text.

Figure 4: a) The $\pi^+\pi^-$ and b) $\pi^+\pi^-\pi^\pm$ mass spectra for the $\pi^+\pi^-\pi^+\pi^-$ final state in the mass range $1.4 \leq$ M($\pi^+\pi^-\pi^+\pi^-$) ≤ 1.5 GeV and with $dP_T \leq 0.2$ GeV. The superimposed histograms are for a $\rho\rho$ (dashed) and $\sigma\sigma$ (shaded) final state. The solid line represents the sum of the $\rho\rho$ (90 %) and $\sigma\sigma$ (10 %) contributions.

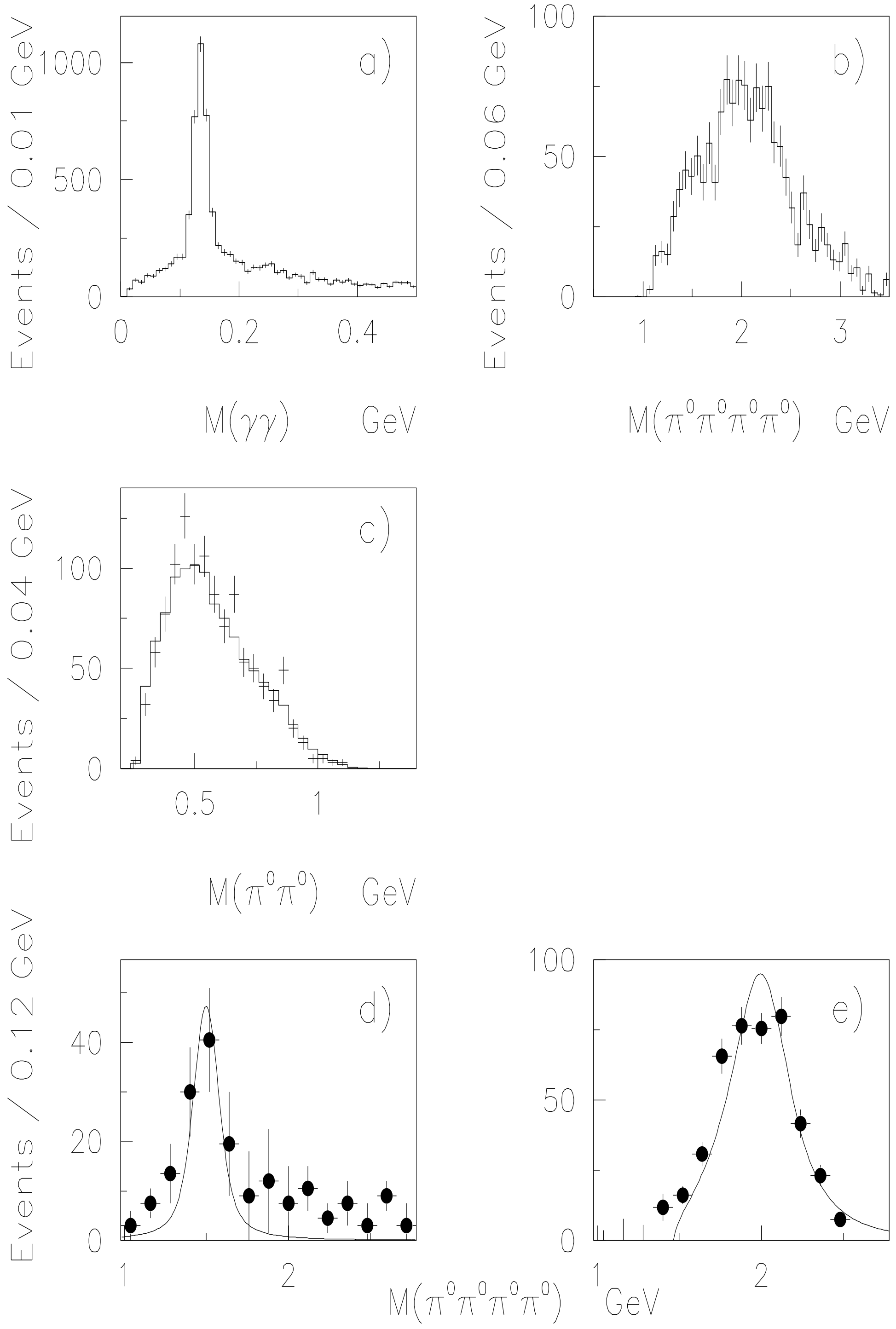
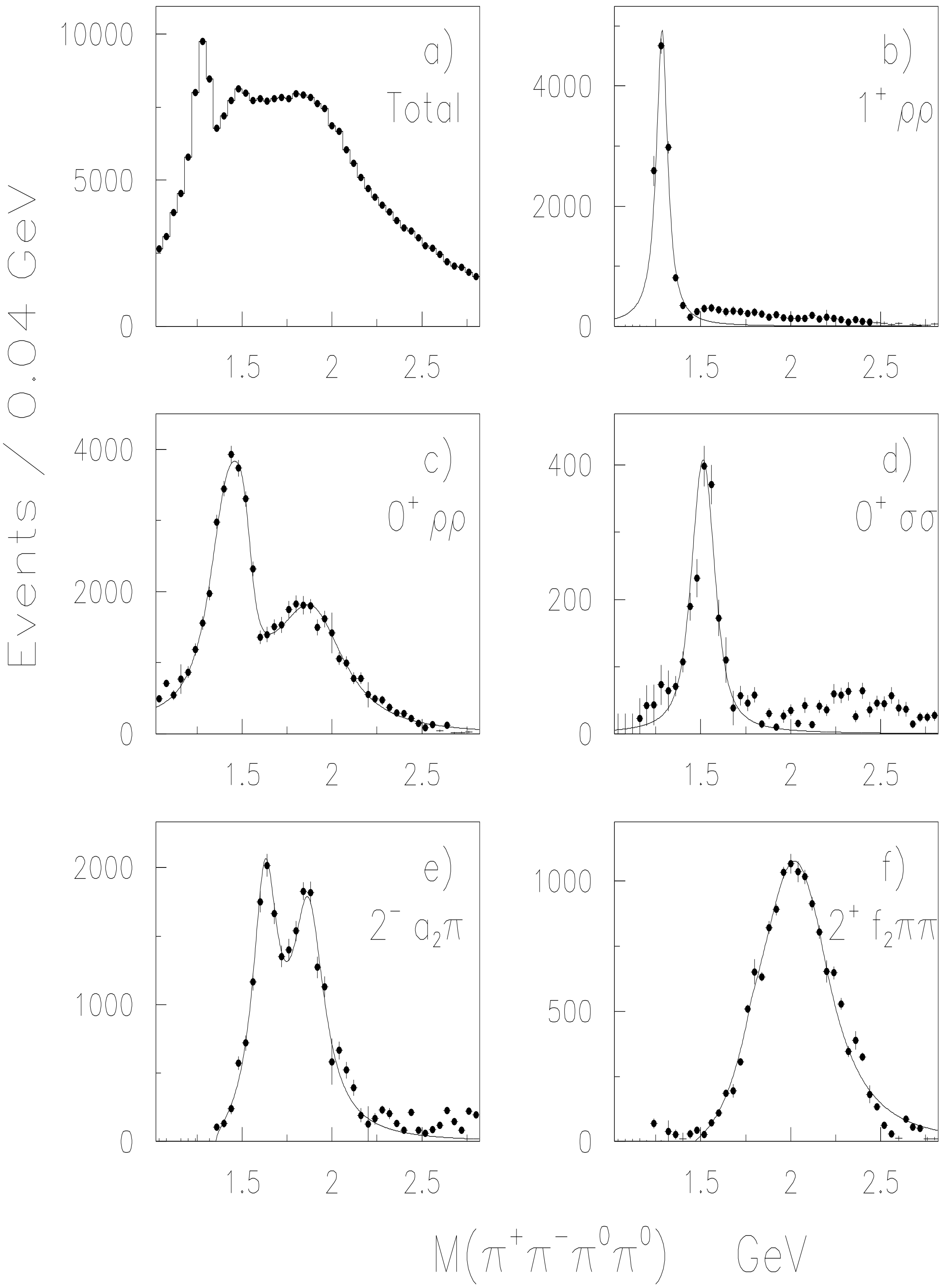

a)
b)
c)
d)
e)
Events / 0.01 GeV
Events / 0.06 GeV
Events / 0.04 GeV
Events / 0.12 GeV
M(γγ) GeV
M(π⁰π⁰π⁰π⁰) GeV
M(π⁰π⁰) GeV
M(π⁰π⁰π⁰π⁰) GeV

Figure 1



a)
Total
b)
$1^+ \rho\rho$
c)
$0^+ \rho\rho$
d)
$0^+ \sigma\sigma$
e)
$2^- a_2\pi$
f)
$2^+ f_2\pi\pi$
Events / 0.04 GeV
$M(\pi^+\pi^-\pi^0\pi^0)$ GeV

Figure 2

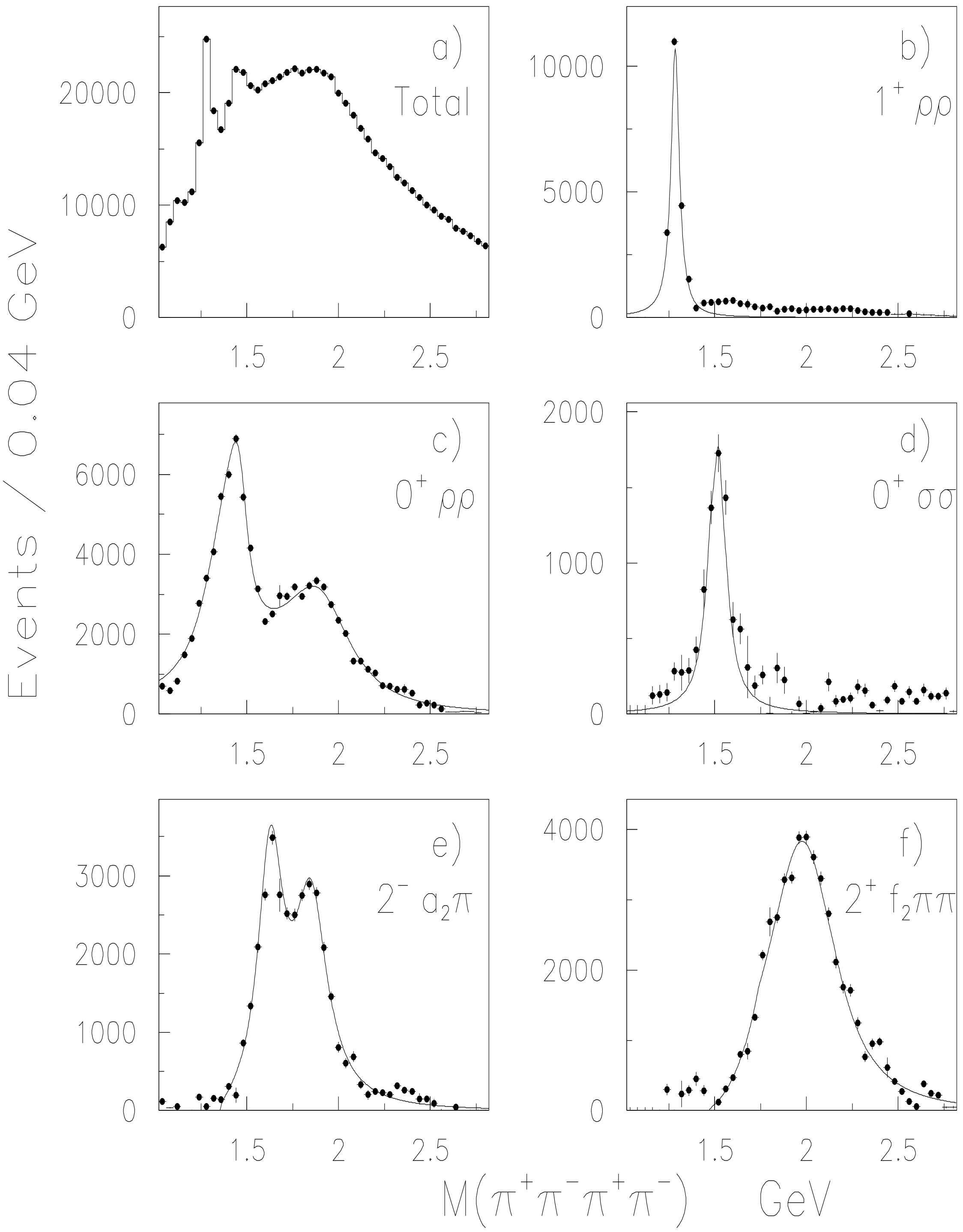
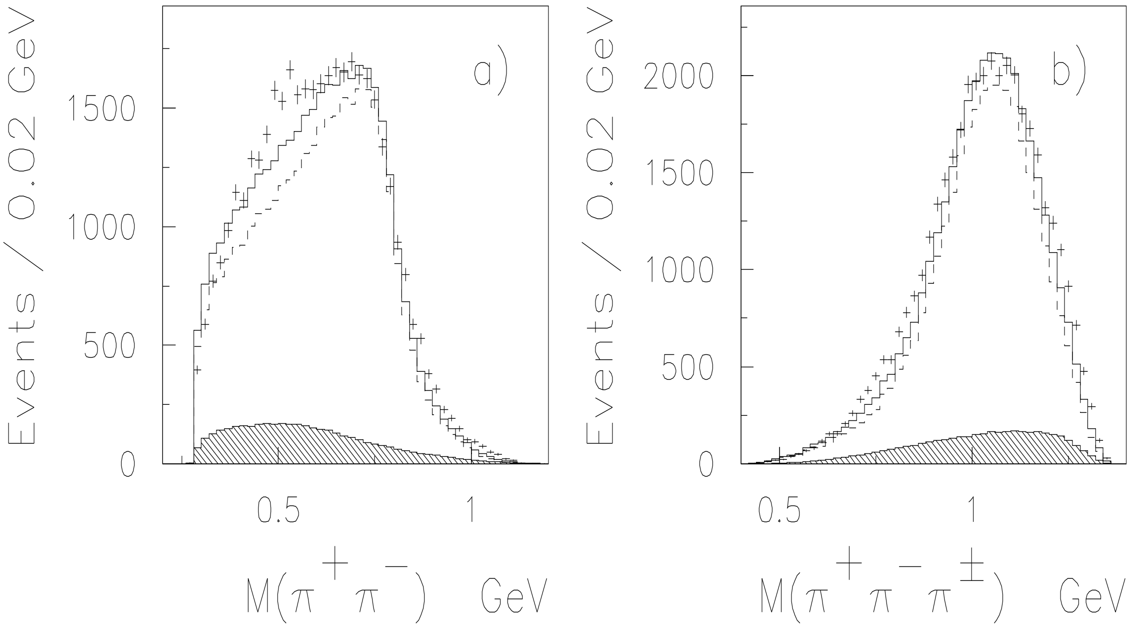

Events / 0.04 GeV
a)
Total
b)
$1^+ \rho\rho$
c)
$0^+ \rho\rho$
d)
$0^+ \sigma\sigma$
e)
$2^- a_2\pi$
f)
$2^+ f_2\pi\pi$
$M(\pi^+\pi^-\pi^+\pi^-)$ GeV

Figure 3



Events / 0.02 GeV
0
500
1000
1500
a)
0.5
1
M(π+π−) GeV
Events / 0.02 GeV
0
500
1000
1500
2000
b)
0.5
1
M(π+π−π±) GeV

Figure 4