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Large isospin mixing in ϕ radiative decay and the spatial size of the $f_0(980)$–$a_0(980)$ mesons

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Abstract

The measured rate for $\phi \to \gamma f_0(980)$ appears to be larger than allowed on rather general grounds. We show that mixing between the $f_0(980)$ and $a_0(980)$, due to their dynamical interaction with the nearby $K\overline{K}$ thresholds, radically affects some existing predictions of their production in ϕ radiative decay. We predict that $\Gamma(\phi \to \gamma f_0)/\Gamma(\phi \to \gamma a_0) \sim 3$; that $\sum(b.r.(\phi \to \gamma f_0) + b.r.(\phi \to \gamma a_0)) < 5 \times 10^{-4}$ with probable individual branching ratios $\sim 2 \times 10^{-4}$ and 0.7×10^{-4}, respectively.

The radiative decay of the ϕ to the enigmatic scalar mesons $f_0(980)$ and $a_0(980)$ has long been recognised as a potential route towards disentangling their nature. Particular interest has focussed on the likelihood that these states contain significant non-$q\bar{q}$ content, specifically being $(u\bar{u} \pm d\bar{d})s\bar{s}$. Such a configuration could either be confined within $R \sim \Lambda_{\rm QCD}^{-1}$ as a "four-quark" state [1,2], or more spatially dispersed into two identifiable colour-singlets: the $K\overline{K}$ molecule scenario [3,4]. In more sophisticated pictures these states could be seeded by an underlying $q\bar{q}$ or compact $qq\bar{q}\bar{q}$ component, influenced by the S-wave $K\overline{K}$ and related thresholds in this mass region [5,6]. Furthermore, significant isospin mixing effects are anticipated (and seen) in the neutral f_0–a_0^0 states due to the nearness of the K^+K^- and $K^0\overline{K}^0$ thresholds [7–9]. In this Letter we note that such isospin mixing effects could considerably alter some predictions in the literature for $\Gamma(\phi \to f_0(980)\gamma)$ and $\Gamma(\phi \to a_0(980)\gamma)$.

The magnitudes of these widths are predicted to be rather sensitive to the fundamental structures of the f_0 and a_0, and as such potentially discriminate amongst them. For example, if $f_0(980) \equiv s\bar{s}$ and the dominant dynamics is the "direct" quark transition $\phi(s\bar{s}) \to \gamma 0^{++}(s\bar{s})$, then the predicted $b.r.(\phi \to \gamma f_0) \sim 10^{-5}$, the rate to $\phi \to \gamma a_0(q\bar{q})$ being even smaller due to OZI suppression [4]. For $K\overline{K}$ molecules the rate was predicted to be higher, $\sim (0.4\text{–}1) \times 10^{-4}$ [4], while for tightly compact $qq\bar{q}\bar{q}$ states the rate is yet higher, $\sim 2 \times 10^{-4}$ [2,4]. Thus at first sight there seems to be a clear means to distinguish amongst them.

In the $K\overline{K}$ molecule and $qq\bar{q}\bar{q}$ scenarios it has uniformly been assumed that the radiative transition will be driven by an intermediate K^+K^- loop ($\phi \to K^+K^- \to \gamma K^+K^- \to \gamma 0^{++}$). Explicit calculations in the literature agree that this implies [2,4,10,11]

$$b.r.(\phi \to f_0(980)\gamma) \sim 2 \pm 0.5 \times 10^{-4} \times F^2(R), \quad (1)$$

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where $F^2(R) = 1$ in point-like effective field theory computations, such as Refs. [2,11]. The range of predicted magnitudes for the branching ratios in Eq. (1) reflect the sensitivity to assumed parameters, such as masses and couplings that vary slightly among these references. By contrast, if the $f_0(980)$ and $a_0(980)$ are spatially extended $K\overline{K}$ molecules (with r.m.s. radius $R > \mathrm{O}(\Lambda_{\mathrm{QCD}}^{-1})$), then the high momentum region of the integration in Refs. [4,10] is cut off, leading in effect to a form factor suppression, $F^2(R) < 1$ [4,12]. The differences in absolute rates are thus intimately linked to the model dependent magnitude of $F^2(R)$.

In any event, one would expect in such pictures that the branching ratio in Eq. (1) will not exceed 2.5×10^{-4}. It is therefore tantalising that the measured rate [13] (which is quoted as an average of the data from Refs. [14,15]) appears to be large:

$$b.r.(\phi \to f_0\gamma) = 3.4 \pm 0.4 \times 10^{-4}. \tag{2}$$

Therefore, precision data on both f_0 and a_0 production, which are expected to be forthcoming soon from DAΦNE, will be most interesting. Whatever the data may be, there are two particular items that we wish to address concerning the current predictions. One concerns the absolute branching ratios, and the second concerns the ratio of branching ratios where, if f_0 and a_0^0 have common constituents (and hence are "siblings") and are eigenstates of isospin, then their affinity for K^+K^- should be the same and so [2,4,11]

$$\frac{\Gamma(\phi \to f_0\gamma)}{\Gamma(\phi \to a_0\gamma)} \sim 1. \tag{3}$$

There are reasons to be suspicious of the predictions in Eqs. (1) and (3). We shall frame our remarks in the context of the $K\overline{K}$ molecule, but they apply more generally.

If in the $K\overline{K}$ molecule one has

$$|f_0\rangle = \cos\theta\left|K^+K^-\right\rangle + \sin\theta\left|K^0\overline{K}^0\right\rangle \tag{4}$$

and

$$\left|a_0^0\right\rangle = \sin\theta\left|K^+K^-\right\rangle - \cos\theta\left|K^0\overline{K}^0\right\rangle, \tag{5}$$

then the branching ratios $\phi \to \gamma f_0(\gamma a_0)$ as found in Ref. [4] can be summarised as follows

$$b.r.(\phi \to \gamma f_0 : \gamma a_0) = (4 \pm 1) \times 10^{-4} \frac{\cos^2\theta}{\sin^2\theta} \left(\frac{g^2_{SK^+K^-}/4\pi}{0.58\ \mathrm{GeV}^2}\right) F^2(R). \tag{6}$$

As shown in Ref. [4], the analytical results of point-like effective field theory calculations (e.g., Refs. [2,11]) can be recovered as $R \to 0$, for which $F^2(R) \to 1$. In contrast to the compact hadronic four quark state, the $K\overline{K}$ molecule is spatially extended with r.m.s. $R \sim 1/\sqrt{m_K\epsilon}$, where ϵ is the binding energy and $F^2(R) < 1$, the precise magnitude depending on the $K\overline{K}$ molecular dynamics, which we shall discuss later. It is clear also that the absolute rate in Eq. (6) is driven by (i) the assumed value for $g^2_{SK^+K^-}/(4\pi) = 0.58\ \mathrm{GeV}^2$, and (ii) the further assumption that the f_0 and a_0 are $K\overline{K}$ states with $I = 0, 1$, hence $\theta = \pi/4$.

There are reasons to question both of these assumptions.

The assumed value $g^2_{f_0K^+K^-}/(4\pi) = 0.58\ \mathrm{GeV}^2$ is consistent with that used in the effective field theory calculations of Refs. [2,11]. However, recent data raise some doubts as to the reliability of this number, and it is not always clear in the literature as to how this coupling is being defined.

We define the coupling of a scalar to two pseudoscalars, as follows. For example, for the $f_0(980)$ which is above threshold for decay into pions,

$$\Gamma\left(f \to \pi^+\pi^-\right) = \frac{g^2_{f\pi^+\pi^-}}{4\pi} \frac{1}{4m_f} \sqrt{1 - \frac{4m_\pi^2}{m_f^2}}.$$

The determination of the actual magnitude of the $g^2_{fK^+K^-}$ coupling requires some care in view of the subtle ways that unitarity can affect the $\pi\pi$ and $K\overline{K}$ couplings when the $K\overline{K}$ threshold is opening, for which a coupled channel analysis is required.

Recently determinations of the couplings of the f_0 to both $\pi\pi$ and to $K\overline{K}$ have been measured in central production by the WA102 Collaboration at CERN [17]. Their data are amenable to a coupled channel analysis and Ref. [17] found

$$\frac{g^2_{f\pi^+\pi^-}}{4\pi} = 0.24 \pm 0.04 \pm 0.05\ \mathrm{GeV}^2,$$

$$\frac{g^2_{fK^+K^-}}{4\pi} = 0.39 \pm 0.04 \pm 0.04\ \mathrm{GeV}^2.$$

(Our convention related to that of Refs. [17,18] is $g^2_{fK\overline{K}}/(4\pi) \equiv g_K \times 2m_f^2$ or $g^2_{fK^+K^-}/(4\pi)\ \mathrm{GeV}^2 \sim g_K$.)

Thus, adopting this value, the predicted rates would be correspondingly renormalised downwards by $\frac{g^2_{fKK}}{4\pi}/0.58 = 0.67 \pm 0.10$ which would make an even greater mismatch with the extant data. Moreover, an analysis of Fermilab E791 [18] data, which studies the $f_0(980)$ produced in D_S decays, even suggests that $g^2_{fK^+K^-}/(4\pi) \sim 0.02 \pm 0.04 \pm 0.03\,\mathrm{GeV}^2$, hence consistent with zero! However, it should be noted that only the $\pi\pi$ decay mode of the $f_0(980)$ has been studied in this experiment and hence the coupling to K^+K^- is only measured indirectly. With such uncertainties in the value of this coupling strength, predictions of absolute rates for $\phi \to \gamma f_0(980)$ or $\phi \to \gamma a_0(980)$ via an intermediate $K\overline{K}$ loop must be treated with some caution.

By contrast, in the ratio of branching ratios this uncertainty is reduced, at least in the case of $K\overline{K}$ molecules for which [4] $\frac{\Gamma(\phi\to f_0\gamma)}{\Gamma(\phi\to a_0\gamma)} \sim 1$. Hence, a significant deviation from equality would appear to be a rather direct elimination of $K\overline{K}$ molecules and, perhaps, other models where a strong affinity of "siblings" to the intermediate K^+K^- state is assumed. This also will be important to test at DAΦNE as, within rather large errors, the results from Ref. [15] in particular suggest that

$$\frac{\Gamma(\phi \to f_0\gamma)}{\Gamma(\phi \to a_0\gamma)} \sim 3.2 \pm 1.8 \tag{7}$$

in contrast to Eq. (3).

In this context, we draw attention to a potentially dramatic effect upon the (relative and absolute) rates for $\phi \to \gamma f_0(980)$ and $\phi \to \gamma a_0(980)$ due to significant isospin mixing in the f_0–a_0^0 system [8]. This effect, which appears to be due to the proximity to the $K\overline{K}$ threshold [7,9] and the differing mass gaps to the K^+K^- and $K^0\overline{K}^0$, could be amplified in ϕ radiative decays that proceed *via* an intermediate $K\overline{K}$ loop [2,4,10,11].

Traditionally in strong interactions isospin has been believed to be a nearly exact symmetry, broken only by the slightly different masses of the u and d quarks and/or electroweak effects. The small difference in mass between $K^\pm$ and K^0 is a particular example. However, the nearness of the ϕ and the $f_0(980)/a_0(980)$ to the K^+K^- and K^0K^0 thresholds causes the relative mass gaps to the charged and neutral thresholds to be substantially different. As a result the dynamics of such strongly coupled $K\overline{K}$ states [3,5,6,19] may be described better in a basis specified by mass eigenstates. Such dynamics would give rise to a violation of isospin and lead to mixing of states with different G-parities.

The possibility of such an effect was suggested long ago in Ref. [7] and has been studied phenomenologically in Ref. [9] and Refs. [20,21]. The former, in particular, has specifically drawn attention to the relation between the existence of $K\overline{K}$ molecular bound states and large violations of isospin. These papers all concentrated on the production of the $f_0(980)/a_0(980)$ by flavoured mesons or photons; in Ref. [4] we proposed that rather clean tests of the mixing could be obtained from their production by gluonic systems, such as the $\mathbb{P}$(Pomeron)-induced production in the central region at high energy: $pp \to pp + f_0(980)/a_0(980)$.

Our analysis showed that new data from the WA102 Collaboration at CERN [16] are already consistent with a significant mixing. Specifically: in (isoscalar) $\mathbb{P}$(Pomeron)-induced production in the central region at high energy, production of the $a_0^0(980)$ comes dominantly from mixing with the $f_0(980)$ such that the f_0–a_0 are not good isospin eigenstates. In the language of the $K\overline{K}$ molecule, at least, this would translate into $\theta \neq \pi/4$ in Eq. (6) and hence to a significant difference in behaviour for $\Gamma(\phi \to \gamma f_0)/\Gamma(\phi \to \gamma a_0)$.

With the basis as defined in Eqs. (4) and (5), the ratio of production rates by $\mathbb{P}\mathbb{P}$(isoscalar) fusion in central production will be

$$\frac{\sigma(\mathbb{P}\mathbb{P} \to a_0)}{\sigma(\mathbb{P}\mathbb{P} \to f_0)} = \frac{1 - \sin 2\theta}{1 + \sin 2\theta}. \tag{8}$$

In Ref. [8] we found this to be $(8 \pm 3) \times 10^{-2}$. Hence, if we assume that the production phase is the same for the two, we obtain

$$\cot\theta = 1.8 \pm 0.2 \quad (\theta = 30^\circ \pm 3^\circ) \tag{9}$$

and hence predict that within this approximation the relative rates will be

$$\frac{\Gamma(\phi \to \gamma f_0)}{\Gamma(\phi \to \gamma a_0)} \equiv \cot^2\theta = 3.2 \pm 0.8. \tag{10}$$

This is far from the naive expectation of unity for ideal isospin states and in remarkable agreement with data (Eq. (7)).

This prediction, Eq. (10), is a rather direct consequence of the isospin mixing obtained in Ref. [8]. In order to use the data to abstract magnitudes of $F^2(R)$, and hence assess how compact the four-quark state is, a definitive accurate value for $g^2_{fKK}/(4\pi)$ will be required.

If for orientation we adhere to the value used elsewhere, $g^2_{fKK}/(4\pi) \sim 0.6$ GeV^2, and impose the preferred θ, then the results of Ref. [4] are revised to

$$b.r.(\phi \to \gamma f_0) + b.r.(\phi \to \gamma a_0) \leqslant (4 \pm 1) \times 10^{-4} \tag{11}$$

and

$$b.r.(\phi \to \gamma f_0) = (3.0 \pm 0.6) \times 10^{-4} F^2(R), \tag{12}$$

$$b.r.(\phi \to \gamma a_0) = (1.0 \pm 0.25) \times 10^{-4} F^2(R). \tag{13}$$

For illustration we cite two models. Barnes [23] developed a simple potential picture of a $K\overline{K}$ molecule, ignoring any short range annihilation and rescattering corrections. This leads to a high momentum cut-off in the K^+K^- loop. Following Barnes' parameterisation, Ref. [4] described the high momentum cut off by a power law, such that the $K^+K^-0^{++}$ vertex form factor $\phi(p) = \mu^4/(p^2+\mu^2)^2$ in which case $\mu \equiv \sqrt{3}/2R$. This led to $R \sim 1.2$ fm, $F^2(R) \sim 0.25$.

However, the predictions are rather sensitive to the assumed details. For example, the authors of Ref. [4] also considered a Gaussian parameterisation for the $K^+K^-0^{++}$ vertex form factor $\phi(p) = e^{-p^2/4\mu_0}$ and $\mu_0 \equiv 3/16R^2$. Barnes' parameters in this case imply that $\mu_0 \sim 0.4$ fm^{-2} and $R \sim 0.7$ fm, in which case the suppression is only some 20%; $F^2(R) \sim 0.8$. In more sophisticated treatments, the role of annihilation involving non-$K\overline{K}$ intermediate states such as $\pi\pi$ and $\pi\eta$ will modify the potential.

If experiment confirms in Eq. (10) predicted ratio, then the individual rates may be used as a measure of $F^2(R)$. Branching ratios for which $F^2(R) \ll 1$ would imply that the $K^+K^-0^{++}$ interaction is spatially extended, $R > \mathrm{O}(\Lambda^{-1}_{\mathrm{QCD}})$. Conversely, if $F^2(R) \to 1$, the system is spatially compact, as in $qq\bar{q}\bar{q}$. If, as preliminary data suggest, the rates are between these extremes, then a qualitative picture may emerge of a compact structure, such as $q\bar{q}$ or $qq\bar{q}\bar{q}$, which spends a sizeable part of its lifetime in a two meson state, such as $K\overline{K}$. Such a picture has also recently been suggested, based on QCD sum rules for an intrinsic $s\bar{s}$ "seed", in Ref. [22].

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