THE MIXING OF THE $f_0(1370)$, $f_0(1500)$ AND $f_0(1710)$ AND THE SEARCH FOR THE SCALAR GLUEBALL

A. KIRK

School of Physics and Astronomy, Birmingham University, U.K.
E-mail: ak@hep.ph.bham.ac.uk

For the first time a complete data set of the two-body decays of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ into all pseudoscalar mesons is available. The implications of these data for the flavour content for these three f_0 states is studied. We find that they are in accord with the hypothesis that the scalar glueball of lattice QCD mixes with the $q\overline{q}$ nonet that also exists in its immediate vicinity. We show that this solution also is compatible with the relative production strengths of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ in pp central production, p$\bar{\text{p}}$ annihilations and J/ψ radiative decays.

1 Introduction

It is now generally accepted that glueballs will mix strongly with nearby $q\overline{q}$ states with the same J^{PC} and that this will lead to three isoscalar states of the same J^{PC} in a similar mass region. In general these mixings will negate the naive folklore that glueball decays would be "flavour blind ".

Lattice gauge theory calculations (in the quenched approximation) predict that the lightest glueball has $J^{PC} = 0^{++}$ and that its mass is in the $1.45 - 1.75$ GeV region. This means that the three states in the glueball mass range are the $f_0(1370)$, $f_0(1500)$ and the $f_0(1710)$.

2 Data from the WA102 experiment

Recently the WA102 collaboration has published [1], for the first time in a single experiment, a complete data set for the decay branching ratios of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ to all pseudoscalar meson pairs (see fig. 1).

A coupled channel fit to this data yields sheet II pole positions of M($f_0(1370)$) = (1310 $\pm$ 19 $\pm$ 10) $-i$ (136 $\pm$ 20 $\pm$ 15) MeV M($f_0(1500)$) = (1508 $\pm$ 8 $\pm$ 8) $-i$ (54 $\pm$ 7 $\pm$ 6) MeV and M($f_0(1710)$) = (1712 $\pm$ 10$\pm$ 11) $-i$ (62 $\pm$ 8 $\pm$ 9) MeV.

The relative decay rates $\pi\pi : K\overline{K} : \eta\eta : \eta\eta' : 4\pi$ are for the $f_0(1370)$: $1 : 0.46 \pm 0.19 : 0.16 \pm 0.07 : 0.0 : 34.0^{+22}_{-9}$ for the $f_0(1500)$: $1 : 0.33 \pm 0.07 : 0.18 \pm 0.03 : 0.096 \pm 0.026 : 1.36 \pm 0.15$ and for the $f_0(1710)$: $1 : 5.0 \pm 0.7 : 2.4 \pm 0.6 :< 0.18\ (90\ \%\ CL) :< 5.4\ (90\ \%\ CL)$

These data will be used as input to a fit to investigate the glueball-quarkonia content of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$.

3 The fit

In the $|G\rangle = |gg\rangle$, $|S\rangle = |s\bar{s}\rangle$, $|N\rangle = |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$ basis, the three physical states can be read as

$$\begin{pmatrix} |f_0(1710)\rangle \\ |f_0(1500)\rangle \\ |f_0(1370)\rangle \end{pmatrix} = \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix} \begin{pmatrix} |G\rangle \\ |S\rangle \\ |N\rangle \end{pmatrix}, \tag{1}$$

where the parameters x_i, y_i and z_i are related to the partial widths of the observed states [2] as given in table 1.

We then perform a χ^2 fit based on the measured branching ratios. The details of the fit are given in ref. [2]. For this presentation the parameter r_3, used in ref. [2], has been set to zero. As input we use the masses of the $f_0(1500)$ and $f_0(1710)$. In this way seven parameters, M_G, M_N, M_S, M_3, f, r_2 and ϕ are determined from the fit. The mass of the $f_0(1370)$ is not well established so we have left it as a free pa-

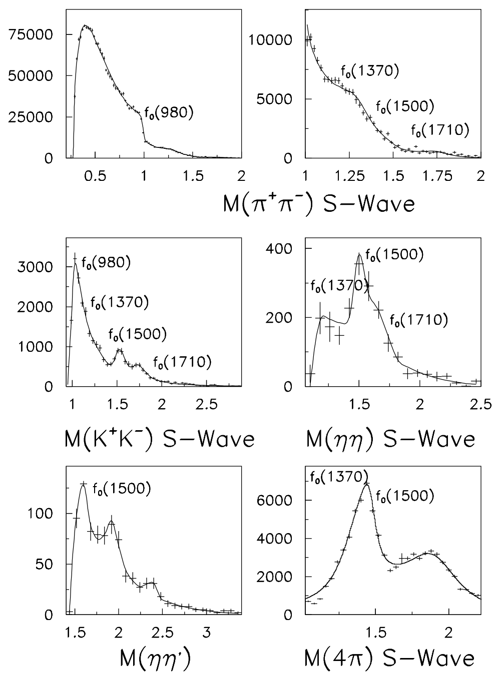

Figure 1. The observation of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ in the WA102 experiment.

Table 1. The theoretical reduced partial widths.

$\gamma^2(f_i \to \eta\eta')$	$2[2\alpha\beta(z_i - \sqrt{2}y_i)]^2$
$\gamma^2(f_i \to \eta\eta)$	$[2\alpha^2 z_i + 2\sqrt{2}\beta^2 y_i + r_2 x_i]^2$
$\gamma^2(f_i \to \pi\pi)$	$3[z_i + r_2 x_i]^2$
$\gamma^2(f_i \to K\bar{K})$	$4[\frac{1}{2}(z_i + \sqrt{2}y_i) + r_2 x_i]^2$

rameter (M_3). The χ^2/NDF of the fit is 13.9/7 and the largest contribution comes from the $\frac{f_0(1710)\to\eta\eta}{f_0(1710)\to K\overline{K}}$ branching ratio which contributes 6.1 to the χ^2.

The mass parameters determined from the fit are M_G = 1446 $\pm$ 16 MeV, $M_S = 1664 \pm 9$ MeV, $M_N = 1374 \pm 28$ MeV and M_3 = 1248 $\pm$ 31 MeV. The output masses for M_N and M_S are consistent with the $K^*(1430)$ being in the nonet and with the glueball mass being at the lower end of the quenched lattice range. The mass found for the $f_0(1370)$ (1256 $\pm$ 31 MeV) is at the lower end of the measured range for this state. The pseudoscalar mixing angle is found to be ϕ = -25 $\pm$ 4 degrees consistent with other determinations.

The physical states $|f_0(1710)\rangle$, $|f_0(1500)\rangle$ and $|f_0(1370)\rangle$ are found to be

$$|f_0(1710)\rangle = 0.42|G\rangle + 0.89|S\rangle + 0.17|N\rangle,$$

$$|f_0(1500)\rangle = -0.61|G\rangle + 0.37|S\rangle - 0.69|N\rangle,$$

$$|f_0(1370)\rangle = 0.65|G\rangle - 0.15|S\rangle - 0.73|N\rangle.$$

Other authors have claimed that $M_G > M_S > M_N$ [3]. This scenario is disfavoured as, if in the fit we require $M_G > M_S > M_N$, the χ^2 increases to 57. In any event, we are cautious about such claims [3] as they are likely to be significantly distorted by the presence of a higher, nearby, excited $n\bar{n}$ state (N^*) such that $M_{N^*} > M_G > M_S$: the philosophy of dominant mixing with the nearest neighbours would then lead again to the "singlet - octet - singlet" scenario that we have found above.

4 Predictions for production mechanisms

Our preferred solution has implications for the production of these states in $\gamma\gamma$ collisions, $\mathrm{p\bar{p}}$ annihilations, in central pp collisions and in radiative J/ψ decays. These are interesting in that they are consequences of the output and were not used as constraints.

4.1 $\gamma\gamma$ production

The most sensitive probe of flavours and phases is in $\gamma\gamma$ couplings. In the spirit of ref. [4], ignoring mass-dependent effects, the above imply $\Gamma(f_1(1710) \to \gamma\gamma)$: $\Gamma(f_1(1500) \to \gamma\gamma)$: $\Gamma(f_1(1370) \to \gamma\gamma)$ = $3.8\pm0.9 : 6.8\pm0.8 : 16.6\pm0.9$. The $\gamma\gamma$ width of $f_0(1500)$ exceeding that of $f_0(1710)$ arises because the glueball is nearer to the N than the S. This shows how these $\gamma\gamma$ couplings have the potential to pin down the input pattern.

4.2 $\mathrm{p\bar{p}}$ production

The production of the f_0 states in $\mathrm{p\overline{p}} \to \pi + f_0$ is expected to be dominantly through the $n\overline{n}$ components of the f_0 state, possibly through gg, but not prominently through the $s\overline{s}$ components. The above mixing pattern implies that $\sigma(\mathrm{p\overline{p}} \to \pi + f_0(1710)) < \sigma(\mathrm{p\overline{p}} \to \pi + f_0(1370)) \sim \sigma(\mathrm{p\overline{p}} \to \pi + f_0(1500))$ Experimentally [5] the relative production rates are, $\mathrm{p\overline{p}} \to \pi + f_0(1370) : \pi + f_0(1500)) \sim 1 : 1$. and there is no evidence for the production of the $f_0(1710)$. This would be natural if the production were via the $n\overline{n}$ component. The actual magnitudes would however be model dependent; at this stage we merely note the consistency of the data with the results of the mixing analysis above.

4.3 Central production

For central production, the cross sections of well established quarkonia in WA102 suggest that the production of $s\overline{s}$ is strongly suppressed relative to $n\overline{n}$. The relative cross sections for the three states of interest here are $\mathrm{pp} \to \mathrm{pp} + (f_0(1710) : f_0(1500) : f_0(1370)) \sim 0.14 : 1.7 : 1$. This would be natural if the production were via the $n\overline{n}$ and gg components.

In addition, the WA102 collaboration has studied the production of these states as a function of the azimuthal angle ϕ, which is defined as the angle between the p_T vectors of the two outgoing protons. An important qualitative characteristic of these data is that the $f_0(1710)$ and $f_0(1500)$ peak as $\phi \to 0$ whereas the $f_0(1370)$ is more peaked as $\phi \to 180$ [6]. If the gg and $n\overline{n}$ components are produced coherently as $\phi \to 0$ but out of phase as $\phi \to 180$, then this pattern of ϕ dependence and relative production rates would follow; however, the relative coherence of gg and $n\overline{n}$ requires a dynamical explanation. We do not have such an explanation and open this for debate.

4.4 Radiative J/ψ decays

In J/ψ radiative decays, the absolute rates depend sensitively on the phases and relative strengths of the G relative to the $q\overline{q}$ component, as well as the relative phase of $n\bar{n}$ and $s\bar{s}$ within the latter. As discussed in ref. [2], based on the mixings found, we expect that the rate for $f_0(1370)$ will be smallest and that the rate of $J/\psi \to \gamma f_0(1500)$ rate will be comparable to $J/\psi \to \gamma f_0(1710)$.

In ref. [7], the branching ratio of $\mathrm{BR}(J/\psi \to \gamma f_0)(f_0 \to \pi\pi + K\bar{K})$ for the $f_0(1500)$ and $f_0(1710)$ is presented. These can be used to show that [2]: $J/\psi \to f_0(1500)$: $J/\psi \to f_0(1710) = 1.0 : 1.1 \pm 0.4$ which is consistent with the prediction above based on our mixed state solution.

4.5 $\pi^- p$ and $K^- p$ production

In these mixed state solutions, both the $f_0(1500)$ and $f_0(1710)$ have $n\bar{n}$ and $s\bar{s}$ contributions and so it would be expected that both would be produced in $\pi^- p$ and $K^- p$ interactions. The $f_0(1500)$ has clearly been observed in $\pi^- p$ interactions and there is also evidence for the production of the $f_0(1500)$ in $K^- p \to K^0_S K^0_S \Lambda$.

There is evidence for the $f_0(1710)$ in the reaction $\pi^- p \to K^0_S K^0_S n$, originally called the $S^{*\prime}(1720)$. One of the longstanding problems of the $f_0(1710)$ is that in spite of its dominant $K\bar{K}$ decay mode it was not observed in $K^- p$ experiments. In ref. [8] it was demonstrated that if the $f_0(1710)$ had $J = 0$, as it has now been found to have, then the contribution in $\pi^- p$ and $K^- p$ are compatible. One word of caution should be given here: the analysis in ref. [8] was performed with a $f_0(1400)$ not a $f_0(1500)$ as we today know to be the case. As a further test of our solution, it would be nice to see the analysis of ref. [8] repeated with the mass and width of the $f_0(1500)$ and the decay parameters of the $f_0(1710)$ determined by the WA102 experiment.

5 Summary

In summary, based on the hypothesis that the scalar glueball mixes with the nearby $q\bar{q}$ nonet states, we have determined the flavour content of the $f_0(1370), f_0(1500)$ and $f_0(1710)$ by studying their decays into all pseudoscalar meson pairs. The solution we have found is also compatible with the relative production strengths of the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ in pp central production, p$\bar{\text{p}}$ annihilations and J/ψ radiative decays.

References

1. D. Barberis *et al.,* Phys. Lett. **B479** (2000) 59.
2. F.E. Close and A. Kirk, Phys. Lett. **B483** (2000) 345.
3. D. Weingarten, Nucl. Phys. Proc. Suppl. **53** (1997) 232; **63** (1998) 194;**73** (1999) 249;
   M. Strohmeier-Presicek, T. Gutsche, R. Vinh Mau, Amand Faessler, Phys. Rev. **D60** (1999) 054010;
   L. Burakovsky, P.R. Page, Phys. Rev.**D59** (1999) 014022.
4. F.E. Close, G. Farrar and Z.P. Li, Phys.Rev. **D55** (1997) 5749.
5. U. Thoma, Proceedings of Hadron 99, Beijing, China 1999.
6. D. Barberis *et al.,* Phys. Lett. **B467** (1999) 165.
7. W. Dunwoodie, Proceedings of Hadron 97, AIP Conf. Series 432 (1997) 753.
8. S. Lindenbaum and R.S. Longacre, Phys. Lett. **B274** (1992) 492.